THE MATHEMATICS TEACHER

Volume XXXII OCTOBER · 1939 Number 6

CONTENTS

PUBLISHED BY THE

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

MENASHA, WISCONSIN : NEW YORK, N.Y.

Entered as second-class matter at the post office at Menasha, Wisconsin. Acceptance for mailing at special rate of postage provided for in the Act of February 28, 1925, embodied in paragraph 4, section 412 P. L. & R., authorized March 1, 1930.

THE MATHEMATICS TEACHER

Official Journal of the National Council
of Teachers of Mathematics

Devoted to the interests of mathematics in Elementary and Secondary Schools

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The National Council has for its object the advancement of mathematics teaching in elementary and secondary schools. All persons interested in mathematics and mathematics teaching are eligible to membership. The membership fee is $2 per year and entitles each member to receive the official journal of the National Council—THE MATHEMATICS TEACHER—which appears monthly except June, July, August and September.

Correspondence relating to editorial matters, subscriptions, advertisements, and other business matters should be addressed to the office of the

THE MATHEMATICS TEACHER
525 West 120th St., New York City (Editorial Office)

Subscription to THE MATHEMATICS TEACHER automatically makes a subscriber a member of the National Council.

SUBSCRIPTION PRICE $2.00 PER YEAR (*eight numbers*)

Foreign postage, 50 cents per year; Canadian postage, 25 cents per year. Single copies 25 cents. Remittances should be made by Post Office Money Order, Express Order, Bank Draft, or personal check and made payable to THE MATHEMATICS TEACHER.

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Volume XXXII Number 6

Edited by William David Reeve

The Craft of Nombrynge

By E. R. Sleight
Albion College, Albion, Michigan

Very few arithmetics appeared in the English language before the sixteenth century. As late as the middle of the fifteenth century every person using even the simplest operations could read Latin, in which language such information was written. Until modern commerce was established, arithmetic was required only for addition and subtraction, and as late as the thirteenth century a student well advanced in science generally knew nothing of division.

The Earliest Arithmetics in English, edited by Robert Steele, and published for the English Text Society, lists only five arithmetics appearing in the English language before the beginning of the sixteenh century. All of these are of the fifteenth century as is shown by the style of English used. Three of them are mere fragments and have received very little attention. The other two, *The Crafte of Nombrynge* and *The Art of Nombrynge*, are more extensive in scope, and it is the purpose of this paper to review the first of these with special emphasis on the methods of operation.

The Crafte of Nombrynge is an interpretation of the *Canto de Algorismo* of Alexander de Villa Dei (1220), and is bound up, together with other scientific treatises, in the British Museum Library. It deals with the science of arithmetic rather than with the art. Each separate idea in the English translation is introduced by one or more lines of the *Canto*, the Latin form being retained. There are only thirty-two pages in which are discussed certain definitions, notation, meaning and use of zero, how to read and write numbers, and the seven rules.

Algorism is the first definition introduced by

HEC ALGORISMUS ARS PRESENS DICITUR; IN QUA TALIBUS INDORUM FRUIMUR BIS QUINQUE FIGURIS.[1]

Then follows in fifteenth century English —*This book is called the book of Algorism, and the book treats the Craft of Numbering, the which Craft is called also Algorism. There was a King of India, the name of whom was Algor, and he made this his craft, and after his name he called it Algorism.*

Very frequently the question and answer method is used. *This present craft is called Algorism, in which we use ten signs of India. Questio. Why ten figures of India? Solucio. For as I have said before they were found first in India by a King of that country that was called Algor.*

It becomes necessary to define certain terms frequently used in this treatise.

[1] Throughout the rest of the article, the Latin quotations will be in capitals and the translations from the fifteenth century English in italic type.

Some numbers are digitus in Latin and digits in English. Some numbers are called articulus in Latin and articles in English. Some numbers are called composite in English.

SUNT DIGITI NUMERI QUI CITRA DENARIUM SUNT.

Here he[2] *tells what a digit is. A digit is a number that is within (or less than) ten, such as 9. 8. 7. 6. 5. 4. 3. 2. 1.*[3]

ARTICUPLI DECUPLI DEGITORUM; COMPOSITI SUNT ILLI QUI CONSTANT EX ARTICULIS DEGITISQUE.

Here he tells what a composite number is, and what are articles. Articles are those which may be divided into numbers of ten and nothing left over, such as twenty, thirty, one hundred, one thousand, etc. A composite number is one that is composed of a digit and an article such as fourteen, fifteen, twenty-five, etc. And so every number that begins (at right) with a digit and ends with an article is a composite number. Thus twenty-five begins with the digit five and ends with the article twenty.

The meaning and place of these numbers is then given, followed by an explanation of the principles of notation with great emphasis on the use of the cypher.

NIL CIFRA SIGNIFICAT SED DAT SIGNARE SEQUENII.

Explain this verse. A cypher means nothing but he[4] *makes the figure that comes after him to mean more than he should if he were absent, as thus 10. Here the number means ten, and if the cypher were away and no figure in front of him it would mean only one, for then he would stand in the first place. And another cypher would mean nothing more unless it keeps the order of the place. A cypher is not a figure significant.*

QUAM PRECEDENTES PLUS ULTIMA SIGNIFICABIT.

The last figure shall mean more than all the others though there were a hundred thousand before it. Thus 17689. The last figure, that is 1, means ten thousand, and all the other figures mean only seven thousand six hundred eighty and nine. And ten thousand is more than all that number. Ergo, the last figure means more than all the number before.

SEPTEM SUNT PARTES, NON PLURES ISTIUS ARTIS; ADDERE, SUBTRAHERE, DUPLARE, DIMIDIARE, SEXTAQUE, DIVIDERE, SED QUINTA MULTIPLICARE; RADICEM EXTRAHERE PARS SEPTIMA DICITUR ESSE.

From this quotation the writer discovers that there are seven operations to be considered: addition, subtraction, duplication, mediation, multiplication, division, and extraction of roots. The remainder of the thirty-two pages of the treatise are devoted to the explanation and use of five of these seven operations, no mention being made of extraction of roots, and division is used only in mediation.

ADDERE SI NUMERO NUMERUM VIS, ORDINE TALI INCIPE; SCRIBE DUAS PRIMO SERIES NUMERORUM PRIMAM SUB PRIMA RECTE PONENDO FIGURAM, ET SIC DE RELIQUIS FACIAS, SI SINT TIBI PLURES.

Here begins the craft of addition. In this craft thou must know four things. First thou must know what is addition. Next thou must know how many rows of figures thou must have. Next thou must know how many diverse cases happen in this craft. And next what is the profit of this craft. As for the first thou must know that addition is a casting together of two numbers into one number. As for the second thou must know that thou shalt have two rows of figures, one under the other as here you may see:

1234
2168.

As for the third thou must know that there are four different cases. As for the fourth thou must know that the profit of this craft is to tell what is the whole number that comes of different numbers.

The four cases are:

1. No partial sum being greater than 9.

[2] "He" refers to the Latin author.
[3] Note the use of the dot.
[4] Note the personal pronoun.

2. At least one partial sum being greater than 9.

3. The case in which at least one partial sum is 10 or a multiple of 10.

4. The case in which there is a cypher in the upper row.

The method of procedure is discussed at length, but it resolves itself into the method used today, the operations being performed from right to left as at present.

A NUMERO NUMERUM SI SIT TIBI DEMERE CURA SCRIBE FIGURARUM SERIES, VT IN ADDICIONE.

This is the chapter on subtraction in which thou must know four things.[5] These four things are identical in all operations, but the definitions are different. *As for the first thou must know that subtraction is the withdrawing of one number from another. As for the second thou must know that there shall be two numbers. As for the third thou must know that there are four different cases. When all digits in the upper row are larger than the corresponding digit in the lower. When at least one digit in the upper row is the same as the corresponding digit in the lower. When at least one digit in the lower row is larger than the corresponding digit of the upper. When the digit in the lower is larger than the corresponding digit in the upper, and the next figures to the left, both above and below are zeros.*

Here again the method is entirely like the method of today, "borrowing" and all. A paragraph is given to *teaching the Craft how thou shalt know, when thou hast subtracted whether thou hast well done or no.* The method is the one now in use, adding the remainder to the subtrahend to give the minuend.

SEQUITUR DE MEDIACION. INCIPE SIC, SI VIS ALIQUEM NUMERUM MEDIARE SCRIBE FIGURARUM SERIEM SOLAM VELUT ANTE.

In this chapter is taught the Craft of mediation, in the which craft you must know four things. As for the first you shall understand that mediation is a taking out of a half of a number out of a whole number, as you would take 3 out of 6. As for the second thou shalt know that thou shalt have one row of figures and no more. As for the third thou must understand that five cases may happen in this craft, and as for the fourth, thou shalt know that the profit of this craft is when thou hast taken away a half of a number to tell what shall be left. If thou wouldst mediate, that is to say take a half of a number, thou must begin thus,—write one row of figures of what number you wish.

POSTEA PROCEDAS MEDIANS, SI PRIMA FIGURA SI PAR AUT IMPAR VIDEAS.

Here he says when thou hast written a row of figures, thou shalt take heed whether the first figure be even or odd in number, and understand that he speaks of the first figure in the right side. And in the right side thou shalt begin in this craft.

QUIA SI FUERIT PAR, DIMIDIABIS EAM, SCRIBENS QUIQUID REMANEBIT:

Here is the first case of this craft which is this, if the figure be even then thou shalt take away from the even figure a half and do away with that figure and set the half in its place. Thus 4. Take a half out of four and that leaves 2. Do away with the 4 and set the 2 in its place.

IMPAR SI FUERIT VNUM DEMAS MEDIARE QUOD NON PRESUMAS, SED QUOD SUPEREST MEDIABIS INDE SUPER TRACTUM FAC DEMPTUM QUOD NOTAT VNUM.

Here is the second case of this craft, the which is this. If the first figure is a number that is odd, the odd number shall not be mediated, but thou shalt mediate that number less one, and write the result as in the first part of this craft. Where thou hast written that, then write such a mark as is here^{w}. *Lo an example, 245. The 5 is odd. Then mediate 4 and replace the 5 by 2. That is to say replace 5 by 2 and make such a mark as w upon his head as thus 242*^{w}*. Then mediate the 4 and the 2. The half is written 121*^{w}*.*

If the first figure is 1, *Thou shalt do away*

[5] See addition.

with the 1 and set there a cypher and a mark over his head. The number 241 is used as an example which by mediation yields 120^{w}. This is merely a special form of the second case.

The third case arises when the second digit is 1. But this again is merely a special form of the case in which the second digit is odd, which is listed as the fourth case and treated thus:

> POSTEA PROCEDAS HAC CONDICIONE SECUNDA: IMPAR SI FUERIT HINC VNUM DEME PRIORI, INSCRIBENS QUINQUE, NAM DENOS SIGNIFICABIT MONOS PREDICTAM.

Here he puts forth the fourth case, the which is this. If it happens the second figure is an odd number, thou shalt take one away from the odd number, the which shall be reckoned as 10. The new number is then mediated, and a 5 is placed over the head of the second digit. As an example the mediated form of 4678 is $233^{5}4$.

> SI MEDIACIO SIT BENE FACTA PROBARE VALEBIS DUPLANDO NUMERUM QUEM PRIMO DIMEDIASTI.

This couplet explains how the operation of mediation may be proved. *The second example was this, 245. When thou hast mediated this number, if thou hast done well, thou shalt have as the mediation this number, 122^{w}. Now double this number, and begin with the left side. Double 1, that shall be 2. Do away with the 1 and set there the 2. Then double the 2 and set there the 4. Then double the other 2, that will be 4. Then double the mark (w) which stands for a half and that shall be 1. Cast that on to 4, and it shall be 5. Do away with the 2 and the mark and you shall have 5, then thou shalt have the number 245, and this was the number which thou haddest when thou began to mediate, as thou mayest see if thou takest heed.*

The same four things are to be known about multiplication as in the operations discussed. Multiplication is defined as *the bringing together of 2 numbers into one number.* The manner in which the two are united is illustrated by an example, thus: *Twice 4 is 8, and this number 8 contains as many times 4 as there are unities in the other number, the which is 2, for in 2 there are 2 unities and so 4 times 2 is 8, as thou knowest well.* The process is thus based on the number of "unities" involved.

In the "craft" of multiplication there are eight operations or types, all based on the product of a digit by a digit. After an elaborate description of this process of multiplying a digit by a digit the following consolation appears. *But nevertheless if thou hast haste to work thou shalt have here a table of figures whereby thou shalt see at once correctly what is the number that comes of the multiplication of two digits. Thus thou shalt work with this figure. As for example if 5 is to be multiplied by 3 look for the 5 in the left side of the triangle, look where the 3 sits in the lower-most row of the triangle. Now go from him upwards to the same row in which the 5 sits. And that number, the which you find here, is the number that comes from the multiplication of the 2 digits.*

1								
2	*4*							
3	*6*	*9*						
4	*8*	*12*	*16*					
5	*10*	*15*	*20*	*25*				
6	*12*	*18*	*24*	*30*	*36*			
7	*14*	*21*	*28*	*35*	*42*	*49*		
8	*16*	*24*	*32*	*40*	*48*	*56*	*64*	
9	*18*	*27*	*36*	*45*	*54*	*63*	*72*	*81*
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*

A quotation of eight lines from the Latin shows how to work in this craft. An example will illustrate. It is desired to multiply 2465 by 232. The problem is thus written $\begin{matrix}2465\\232\end{matrix}$. Then follows *Thou shalt begin to multiply from the left side. Multiply 2 by 2 and set the result over the head of 2, then multiply the same upper 2 by 3 of the lower number, thus thrice 2 shall be 6. Set the 6 over the head of 3, then multiply the same upper 2 by the 2 that stands under it, that will be four. Do away with the upper 2 and replace it by 4. The upper row will then*

be 464465, the 465 remaining unchanged as no operation was performed upon these digits. Then follows a process called antery which means that the problem is now written thus: *464465* The antery refers to
232
the change produced on the lower line, the whole being moved one place to the right, the 2 which formerly appeared under the fourth digit from the right now appears under the third. Then as before the 4 of the upper line multiplies the digits of the lower line in succession, beginning with the left. The product is now placed above the lower digit, as in the case of the product of the 4 and 2, the 8 is placed above the 6. In case the product is a composite number, such as the product of 4 and 3 the units digit is placed above the corresponding digit of the lower line, while the tenths digit is placed above the topmost digit in the previous column; in this case above the 8. In the last multiplication in each antery, the units digit replaces the number above it in the upper line, while the tenths digit takes its position in the previous column. The result of this multiplication may thus be written:

1
82
464865
232

Another antery and the problem becomes
1
82
464865, which on multiplication yields
232

11
121
828
464825
232

Another antery followed by the necessary multiplication and our problem takes this form, in which the multiplier is omitted.

11
110
1211
8285
464820

The sum of these numbers yields the product. The addition may be performed in the usual manner, but we are told to: *begin with the left,—now draw all these figures down together, as thus 6. 8. 1.*[6] *and 1; that whole is 16. Do away all this number save 6. Let him stand alone and set 1 over the head of 4 toward the left side, then draw onto 4, that will be 5. Do away with that 4 and 1 and set there the 5. Then draw 4. 2. 2. 1 and 1 together, that will be 10. Do away all that and write 0 and set the 1 over the next figure to the left side, the which is 6. Then draw that 6 and 1 together and that will be 7. Now do away with the 6 and set there the 7. Then draw 8. 8. 1. and 1 together and that will be 18; do away all the figures that stand over the head of the 8, and let 8 stand still, and write the 1 over the next figures head to the left, which is 0. Then do away with the 0 and set there the 1, the which stands over the head of 0. Then draw the 2, 5, and 1*[7] *together, that will be 8. Then do away all that and write the 8. And then thou shalt have this number, 571880.*

The above process might be thus indicated:

4
16
10
18
80
———
571880

It will be noted that addition here is performed from left to right, and not from right to left as defined in the previous discussion concerning addition. It leads one to wonder why the two methods are used. It is quite probable that this second plan is introduced to be consistent with the order of multiplication as here used.

Eight cases in multiplication are recognized, depending upon the types of products, or upon the form of the problem.

1. When the product is an article.

[6] Here written without the dot. But I have used it as was the case in naming the digits.

[7] Here the comma is used to separate the digits.

2. When the product is a composite number.

3. When the product is a digit.

4. When the lower digit multiplies the upper digit directly above it.

5. When the lower digit multiplies the upper digit, and that upper digit is not directly above it.

6. If it happens that a zero stands right over the figure by which you multiply.

7. If it happens that the figure by which you multiply is a zero.

8. If there are several zeros in the upper row.

Each of these cases is treated at length, but all of them reduce to the method outlined in the given example.

Although division and extracting roots are listed, among the seven operations, no mention is made of the latter, and division is mentioned only in the process of mediation. One is led to wonder why this omission occurs, since the original poem discusses these operations in detail. It is highly probable that some of the original manuscript has been lost, which would account for the omission of these topics.

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Elementary Approximate Computation

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ACCORDING to several curriculum reports,[1] a desired objective of mathematical instruction is to develop sound judgment in the use of approximate data in computation. Although a dozen years have elapsed since this topic was first recommended as a worthy objective, few textbooks present a workable program of approximate computation[2] and many teachers are not too well acquainted with the theory involved.

The writer believes that much substantial progress in approximate computation technique can be accomplished if the subject is presented *as a whole*, especially in the professional courses for prospective teachers. A certain order of presentation of concepts and a specific vocabulary seem necessary to explain clearly and discuss satisfactorily problems involving approximate computation. Nor is it sufficient merely to learn the definitions of terms since there must be *continual* and *varied practice using these terms* in the various operations in which approximate computation may arise if real learning is to be accomplished. The maxim, "Learn by doing," seems to the writer to be especially helpful in gaining a mastery of this topic. In line with these thoughts, a usable body of theory and exercises have been prepared. It is presented with the hope that teachers will find it helpful both to learn about the elementary theory of approximate computation and to teach it to their secondary school pupils by using it as text material.[3]

INTRODUCTION

Numbers were invented by early man to describe the size of a group of separate things such as eight sheep, or to indicate the position of an object in a series such as the third mountain. Numbers fitted these separate things exactly because each object could be made to correspond perfectly with each number. When early man attempted to describe lengths, areas, volumes, or weights by his invented numbers, he used his numbers in an entirely different way. Hogben says, "There is a very big difference in the way we use numbers when we say that there are 365 days in a year and 24 hours make a day. When people began to divide up the day by the position of the sun's shadow they started to use the old distinct numbers in a way which was quite novel. An hour is not separated by a natural event like the period of moisture between one dry season or another, or the succession of lunar aspects between two full moons. Hours and minutes only correspond with *measurements* on a scale which we can use with greater or less precision according to the care we take in making the scale and in

[1] a. *The Reorganization of Mathematics in Secondary Education.* A report by the National Committee on Mathematical Requirements. (The Mathematical Association of America, Inc., 1923), p. 6.

b. *Curriculum Problems in Teaching Mathematics.* Second Yearbook. National Council of Teachers of Mathematics. (New York, Bureau of Publications, Teachers College, Columbia University, New York, 1927), p. 131.

c. *Selected Topics in the Teaching of Mathematics.* Third Yearbook. National Council of Teachers of Mathematics. (New York, Bureau of Publications, Teachers College, Columbia University, New York, 1928), pp. 141–194.

d. *The Place of Mathematics in Secondary Education.* A Preliminary Report of The Joint Commission of The Mathematical Association of America and The National Council of Teachers of Mathematics. (Ann Arbor, Michigan, Edwards Brothers, Inc., 1938), pp. 85, 104, 106, 131.

[2] Boyer, Lee Emerson. *A Professional Study of Elementary Approximate Computation.* Unpublished Research Project. (State College, Pa. School of Education, Pennsylvania State College, 1938), pp. 13–14.

[3] Bound reprints of this article may be secured from THE MATHEMATICS TEACHER or from the writer at a small cost.

observing the points."[4] Later on the same author says, "The dual use of numbers for counts and estimates [measurements] has been the source of continual misunderstanding between the practical man and the mathematician. It [this dual use of numbers] produced *the first crisis in the history of mathematics*."[5]

Though the race has been handicapped by this "crisis" in numerology, present day writers and computers may well end the crisis by clearly distinguishing between the two *uses* of numbers. This can be done most easily by clearly distinguishing between *kinds* of numbers. But before doing this it is necessary to clarify, or standardize, the use of some words and practices associated with number usage.

Correctness

It has been seen that numbers were invented to describe separate things and that they fitted these separate things perfectly, but that this perfect fit, or correspondence, could not be accomplished when numbers were related to measurements. When measuring a length, for example, the *nearest* division on the tape that corresponds to the length is chosen as the *correct* measurement. If a boy says his sled is 5.8 ft. long he means that the length of his sled lies between the tape marks of 5.75 ft. and 5.85 ft. If his attention is called to the hundredth marks on the tape, he may say his sled is 5.82 ft. long. Now, if the boy is given a magnifying glass and asked to use it to measure the length of the sled, he would probably agree that his sled is not exactly 5.82 ft. long, but that its length lies between 5.815 ft. and 5.825 ft. This difference between the length of an object and a corresponding number point on a measuring scale is not unusual. If a balance between two objects on one pair of scales is believed to be established, disillusionment is encountered when a more sensitive balance is employed. Thus it may be said that *all measurements are only approximations.*

Definition of a Correct Number

A number is said to be correct if the difference between the number and the quantity it represents is less than one half of the unit in terms of which the measure is expressed. For example, 25.6 ft. is the *correct* expression in tenths of a foot, for a length lying between the tape marks of 25.55 ft. and 25.65 ft.

Exercises:

1. Measure the length of the teacher's desk in your room to the nearest inch; to the nearest tenth of a foot; to the nearest hundredth of a foot.

2. What is the enrollment of your school to the nearest hundred; to the nearest fifty?

Measured Numbers Are Approximate Numbers

By this time it is clear that many of the numbers that are met and used in everyday life have been determined by a process of measuring and thus might be called "measured" numbers. It was noticed earlier that the "measured" number the boy gave to represent the length of his sled was, at best, a close approximation. If it is desired to find the surface area of a certain pavement it is necessary to know its width. The width is measured and found to be 4.62 ft.; but this is the pavement's width at only one point. At another point it is found to be 4.58 ft. At still another point the width may be correctly expressed as 4.60 ft. Considering all of these measurements and remembering that only one expression for the width of the pavement is desired, it may correctly be said that the width of the pavement is 4.6 ft. Although the pavement is as carefully built as most pavements are, its width actually varies between 4.58 ft. and 4.62 ft. In addition to irregularities of construction and the difficulties of matching a length to a number mark on the tape, the process of measuring often

[4] Hogben, Lancelot—*Mathematics for the Million* (New York, W. W. Norton and Company, 1937) p. 48.

[5] *Ibid.*, pp. 72–73. Italics are the writer's.

involves lifting the tape and matching ends to a mark. Every time this is done there is danger of an imperfect match. In the light of all of this, it may be said that every "measured" number is an "approximate" number.

Counted or Exact Numbers

While a snowfall of 9 in. actually may be anywhere between 8.5 in. and 9.5 in. deep, a class of 23 pupils consists of exactly 23 persons, no more and no less. Similarly, a flock of 10 sheep consists of exactly 10 animals. These exact numbers may be called "counted" numbers[6] to contrast "measured" numbers, which are always approximations. These two kinds of numbers are sometimes not easily distinguished. Consider the phrase "four pounds of butter." Is this an approximate (measured) number or an exact (counted) number? The word *pounds* suggests the measurement of weight. Thus, it is very likely that "four" is an approximate number since the amount of butter may weigh four pounds plus or minus several ounces. However "four" used to enumerate packs or cartons of butter, meaning four packages of approximately one pound each, is an exact number since it is obviously determined by counting the packages. Perhaps the phrase *expression of measures* as contrasted with *enumeration of objects* will serve as a "key" to distinguish between the two kinds of numbers—the measured numbers, and the counted numbers.

Theoretical numbers like, 75 per cent, $3\frac{1}{2}$, $\frac{2}{3}$, and expressions of dollars and cents are usually classified as exact numbers. In this case the $3\frac{1}{2}$ or fraction $\frac{2}{3}$ is being used to describe the ratio between the numbers of objects or individuals comprising two groups. If a storekeeper is able to sell only 100 loaves of bread from a total of 125 loaves before it gets too stale, the number of loaves sold will be exactly $\frac{4}{5}$ of the number of loaves purchased from the bakery. However, if a boy is halted by rain in the painting of a fence 125 feet long when he has pointed only 100 feet, we cannot say that $\frac{4}{5}$ of the length of the fence is painted in exactly the same sense, for the measurements of 125 feet and 100 feet, like all measurements, are fallible beyond a certain point. Consequently common fractions should be used to express exact fractions, and decimal fractions to express approximate ratios. Decimals furnish a beautiful means of expressing just how much is known and how much is not known.[7]

Exercise:

In the following list indicate the measured (approximate) numbers by placing an "M" after the expression. Likewise, indicate the counted (exact) numbers by placing a "C" after the number.

1. 16 cows
2. 10 books
3. 12 feet (linear)
4. 130 miles
5. 27 blocks (objects)
6. 6 bushels of grain
7. 12 blocks (city distance)
8. 25 cents
9. 6.7
10. 25%
11. 10 teaspoonfuls of medicine
12. 2 dozen eggs
13. 3 pounds of butter
14. $1.31
15. 2 sq. ft. of sheet metal
16. 9 bottles
17. 6 feet 4 inches tall
18. 7 dollars
19. 2.4
20. $2.51

[6] In one sense "counted" numbers may be regarded as "measured" numbers. For example "10 men" indicates the measurement of the group. However, for clearness of teaching effect, the writer is willing to make this somewhat artificial classification.

[7] See Hogben, *op. cit.*, pp. 76–77, for additional thoughts along this line. Also Bond, E. A. *Arithmetic for Teacher Training Institutions:* Contributions to Education No. 525. (New York, Bureau of Publications, Teachers College, Columbia University. 1934), pp. 156, 166, 196 and 200.

"Round" Numbers or Estimates Are Approximate Numbers

Very often there are needs requiring general impressions concerning numbers. For example, the newspaper might announce the attendance at the Fair to be 2,000 people. Or, the approximate distance from the earth to the sun is 93,000,000 miles. Neither of these numbers is exact. They are merely *estimates*. If a man judges the length of a barn he might say to himself, "I believe that barn is somewhere between 75 ft. and 85 ft.," and then *estimate* the length to be 80 ft. The phrase "80 ft." in this case may be called a "round" number or an "estimate," but both obviously are also approximate numbers.

Exercise:

Place an "R" after the numbers in the following list which seem to you to be "round" numbers. Place an "E" after the numbers that seem to you to be exact.

1. 2400 people
2. 1731 cars
3. $800, an item in a family budget to cover expenses for sending a child to college.
4. 50 cars
5. 132 people
6. $8.40, amount on a store bill.

Significant Figures

Consider the number 222. Although all the digits in this number are identical, each has a different value. If $222 is to be deposited in a bank, it would be serious if the clerk, by error, recorded a 1 for the 2 farthest to the right, but much more serious if the 2 farthest to the left were changed to a 1. This idea of varying value for different positions in a number is one of the greatest inventions man ever made, but it also gives rise to some confusion in thought about numbers.

When recording the distance from the earth to the sun, six zeros are used merely to push the 93 into the millions position. Thus, the zeros in 93,000,000 mi. merely locate the decimal point. These zeros are said to be non-significant for this reason. In the expressions $70.40 or 7.0 mi. the zeros tell something specifically in each case. In the first case, they tell exactly how many dollars and how many cents were under consideration. In the second case, the zero tells that the distance was not measured merely to the nearest mile but actually to the nearest tenth of a mile. These zeros are said to be significant.

The following might serve as guiding rules in determining which figures are significant and which are non-significant.

1. All of the non-zero digits are always significant whenever used.

2. Zeros occurring between non-zero digits are always significant.

3. Final zeros of a measured whole number may be *significant.*

In the number .002 the zeros are not significant because they are used merely to locate the decimal point, but in the number .20 the zero is significant for it is not used to locate the decimal point. Similarly, there are five significant figures in each of the numbers 302.06, 362.20, 0.0072689, 500.00 and 72346 but only two in the number 93,000,000 which states the approximate distance from the earth to the sun. When the velocity of light is expressed as 186,000 mi. per second, it is expressed with three significant figures and indicates that this is the velocity to the nearest 1000 mi. per second. The ratio of the circumference of a circle to its diameter is expressed with five significant figures when written 3.1416.

Careful study of the topic reveals that the big difficulty with respect to ability to distinguish between significant and non-significant figures rests with the zeros to the right of the last non-zero figure in a number. Within the past few years at least four practices have been advocated in an effort to remove this difficulty. In this article the method suggested by Bakst[8] will hereafter be used, while in the

[8] Bakst, A. *Approximate Computation.* Twelfth Yearbook, National Council of Teach-

footnotes below a brief explanation of another method and references describing the other two methods will be given.[9] Bakst suggests writing the doubtful non-significant zeros, which appear to the right of the last significant figure in a number, smaller than the significant zeros. Example: 52_0 means that 52_0 contains two significant figures. 520 contains three significant figures.

Exercises:

Determine the number of significant figures in the following numbers.

1. 62	9. 2456
2. 83	10. .0765
3. 607	11. 180
4. 12003	12. 39_0
5. 9.0	13. 37_{00}
6. 0.0004	14. 70
7. .020	15. 7_0
8. 8.00	16. 1250_{00}

Rounding-off Numbers

Following is a quotation from Richardson: "Sometimes we are furnished with numbers recording measurements that are given with greater accuracy than we can use, or care to use. A number is rounded off by dropping one or more digits at the right. When the digit dropped is more than 5, increase the preceding digit by unity; when it is less than 5, retain the preceding digit unchanged. When 5 is dropped, the usual procedure is to *increase* the preceding digit by unity if it is an odd number, and leave it unchanged if it is an even number. The errors due to rounding off, if the rule is followed consistently, will in the long run tend to compensate each other."[10]

The soundness of this practice is based on the fact that considering a rather large array of numbers to be rounded off, the digits to be dropped are rather evenly divided between 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. For example, suppose the summation of a large array of *three-decimal-place* numbers is being considered. The number of addends ending in 7 will obviously closely approximate the number of addends ending in 4 or in 5, or 9, etc. Now, if it is desired to round off the addends to *two-decimal-place* numbers, the obtainable sum will be deflated or inflated when the ending digit of the addends is less or greater than five. Thus, when the numbers ending in 1, 2, 3, 4, are rounded off without increasing by one the preceding digits, the obtainable sum will be deflated. But, this deflation of the obtainable sum will be balanced by the second digits of the addends ending in 6, 7, 8, and 9 which are increased by one. Now, if it is wished to maintain a balance between inflation and deflation of the obtainable sum it will be necessary to boost the digit preceding the 5 when this preceding digit is odd. If more than a single digit is dropped, 5 in the above statement should be replaced by 50, 500, etc. Thus 3451 rounded to a number having two significant figures would be 35_{00}. Since the number of even digits (including zero) equals the number of odd digits, this practice will tend in the long

ers of Mathematics. (New York, Bureau of Publications, Teachers College, Columbia University. 1937), p. 41.

[9] The so-called scientific notation breaks the number into two factors, one of which is expressed as a power of 10, to distinguish between significant and non-significant zeros appearing on the right of a number. All of the significant figures appear in the factor raised to the first power. The non-significant zeros are expressed by some power of 10. In this form of notation it has become customary to place a decimal point between the first and second significant figures.

Examples:

1. *If the number 42,000 is correct to thousands and thus features 3 significant figures, we would write it in scientific notation,* 4.20×10^4.

2. *If 93,000,000 miles is correct to the nearest million miles we would write it* 9.3×10^7.

Two other suggested means of distinguishing between significant and non-significant zeros are made by Helen Walker in *Mathematics Essential for Elementary Statistics* (New York, Henry Holt and Co. 1934), p. 8 and by, Cooley, H. R., Gans, D., Kline, N. and Wahlert, H. E. in *Introduction to Mathematics* (New York, Houghton Mifflin Co. 1937), p. 106.

[10] Richardson, C. H.—*An Introduction to Statistical Analysis* (New York, Harcourt, Brace and Co. 1935), p. 10.

run neither to inflate nor deflate the obtainable sum. When integral numbers greater than 10 are rounded off it is necessary to add zeros to retain the place value. For example, if 3468 is rounded off to two significant figures, it will become 3500 and not 35. Similarly 3,850,000 rounded off to the nearest hundred thousand becomes 3,800,000.

"Rounded" Numbers Are Approximate Numbers

The method by which "rounded" numbers are created places them automatically under the heading, approximate. Thus the process of measuring is not the only one that gives rise to approximate numbers. Sometimes it is necessary to change exact numbers such as $\frac{1}{3}$, $\frac{2}{3}$, or $\sqrt{2}$ to decimal form. Thus an approximation is introduced which will yield only approximate results. A cube whose edge is $\sqrt[3]{2}$ feet contains exactly 2 cu. ft. If the cube root of 2 is extracted approximately and the result cubed, the calculated volume will not be 2 cu. ft. In view of this wide spread influence of approximate numbers, it is no wonder that Schaaf said, "It is probably safe to assert that one-half the time usually spent on computation is wasted, due to the retention of more figures than the precision[11] of the data warrants."[12]

Exercises:

1. Round the following numbers to three significant figures:

a. 78.65	f. 1.2352
b. 2.061	g. 24.457
c. 8028	h. 3458
d. 2335	i. 3.005
e. 67340	j. .00003746

2. Round off the following numbers to the nearest hundred thousand:

a. 1,769,000	c. 6,249,000
b. 3,850,000	d. 8,450,000

[11] For meaning of *precision* see topic, "Same Degree of Precision" on page 255.

[12] Schaaf, W. L.—*Mathematics for Junior High School Teachers* (New York, Johnson Publishing Co. 1931), p. 9.

Limits of Approximate Numbers

No matter how an approximate number is determined, it represents a quantity, or quantities, within certain limits. It is essential to know the limits of an approximate number in order to be intelligent in the use of it. The definition of a correctly expressed approximate number was given on page 250. It is noted that the limits of an approximate number can easily be found by adding and subtracting .5 of the unit in terms of which the measure is significantly expressed to and from the given number. Examples: *1. What are the limits of the approximate number 58 ft.?* Since this measure is expressed in feet, .5 of this unit of measure is .5 of a foot. By adding .5 to and subtracting it from 58 ft., the limits of 58 ft. are found to be 58.5 ft. and 57.5 ft. *2.* Similarly the limits of 3.35 are 3.355 and 3.345. The limits of 2600 are 2650 and 2550

Exercises:

1. Between what two numbers do the following measurements lie?

a. 32.8	e. 1800 (people)
b. 67.3	f. 30 cars
c. 6.5	g. 5000
d. 128	h. 800

2. Explain the difference between

a. 67.4 and 67.40 and

b. 0.62 and 0.620

3. Why is it inaccurate to write 87.2 as 87.20?

Possible Error and Relative Error

Sometimes it is advantageous when dealing with approximate numbers to recognize the possible error. Since a distance of 25 in. is understood to lie between 24.5 and 25.5 in., it is obvious that the possible error of this measurement, 25 in., is .5 of an in. Similarly the possible errors of 23.3 and 0.03 are .05 and .005 respectively.

When the measurements of two or more distances are to be compared it can best be done by comparing the relative errors of the measurements. *The relative error of a*

measurement is the ratio of the possible error to the measurement itself. This can be expressed in fraction form as *relative error.* If 25 is the given measurement the relative error would be $\frac{.5}{25.0}$. Reduced to the lowest terms this would be 1/50 or 2 per cent. Sometimes the relative error of a measurement is more useful when expressed as a fraction and at other times the per cent form seems more useful.

Exercises:

1. What is the possible error of each of the following measurements?

a. 6 b. .02 c. 1.345

2. What is the relative error of each of the following measurements?

a. 7 b. 236 c. 0.4 d. 0.40

3. Rank the following numbers as to the size or their possible errors.

16 1.6 1.60 16oo and .002

Same Degree of Accuracy

If an error of 1 foot is made in measuring a plot of ground 132 feet long, the relative error is 1/132 of the distance measured. If on the other hand an error of 1 foot is made in measuring a mile, 5280 feet, the relative error is 1/5280 of the distance measured. Comparing the two fractions one notes that the latter measurement features much greater accuracy since the error involved was only 1/5280 of the total distance, while in the first measurement the error involved was 1/132 of the total distance. By finding the relative errors of the measurements 2, 23, 345, 2.2, 3.22 and .002 determine which have about the same degree of accuracy. Do the number of significant figures give a hint as to "same degree of accuracy"?

Exercises:

Underline the pairs of numbers that feature "same degree of accuracy."

1. 8 ft. and 96 ft.
2. 3.14 ft. and 201 ft.
3. 2.78 ft. and 642 ft.
4. 8.34 ft. and 28.2 ft.
5. 23 ft. and 1o ft.
6. 46 ft. and 9o ft.
7. 2.8 ft. and 7 ft.
8. 1742 ft. and 823.2 ft.
9. 6702 ft. and 10.80 ft.
10. 75 ft. and 22 ft.
11. 607 ft. and .340 ft.
12. 246 in. and 30.0 in.
13. 24 ft. and 306o ft.
14. 25oo ft. and 340 ft.

Same Degree of Precision

Numbers having equal possible errors feature the same degree of precision. The following pairs of numbers feature the same degree of precision:

a. 36oo and 13oo c. 14 and 16
b. 25.3 and 78.2 d. .0034 and 24.3756

The numbers in pair a. are each measured to the nearest hundred, those in b. to the nearest tenth, those in c. to the nearest unit, and those in d. to the nearest ten-thousandth. It follows that the smaller the unit of measure in terms of which a measurement is expressed, the more precise is the measurement. The measurement 12.24 ft. is more precise than the measurement 13.2 ft. because in the former case the tape was read to the nearest *hundredth* of a foot while in the second case it was read only to the nearest *tenth* of a foot.

Exercise:

Rank the following numbers as to precision. Let the first have the least precision and the last the greatest precision.

1. .003 *2.* 24 *3.* 26o *4.* 2.43
5. 8162.2 *6.* 125ooo

Multiplication of Approximate Numbers

Suppose it is desired to find the area of a plot whose dimensions are 24.66 and 79.73 ft. Ordinarily, one would multiply the length, 79.73, by the width, 24.66, and find the area to be 1966.1418 sq. ft. (Notice that this area is expressed to ten-thousandths of a square foot.) It is reasonable to believe that the length of this plot may have been almost as little as 79.725 or nearly as great as 79.735. In either

case, the length to the nearest hundredth of a foot would have been recorded as 79.73 ft. Similarly, the actual width of the plot might have varied between the limits 24.655 and 24.665 and still have been recorded as 24.66 to the nearest hundredth of a foot. Multiplying the shortest possible length, 79.725, by the shortest possible width, 24.655 indicates an area of 1965.619875 sq. ft. Multiplying the longest possible length, 79.735, by the longest possible width, 25.665, indicates an area of 1966.663775 sq. ft. A comparison of the least possible area, the area as ordinarily computed, and the largest possible area, reveals a striking similarity as far as the first four figures on the left end of the product are concerned. It is seen that even here the fourth figure to the right is somewhat doubtful. If one tolerates the variation in this single digit's place, that is, admits its possible non-correctness and carries it along in the product, certainly he does not care to be concerned with the following digits whose values are inconsequential as compared with the above-mentioned figure in doubt. It has come to be generally understood that the results of multiplication involving approximate numbers should not be expressed with more significant figures than are contained in the factor which has the fewer significant figures. Since the length and width of the above example were originally expressed with four figure accuracy the area would be rounded off to four significant figures and called 1966 sq. ft. *approximately.*

Since no more significant figures are retained in the product than are in the factor featuring the fewer significant figures, it is useless to carry six significant figures in one factor and only two in the other. In such cases we should round off the factors so that there are the same number of significant figures in one factor as in the other.

The rounded-off product *may or may not be correct.* For example $11.7\times12.3=143.91$. This product rounded-off according to our rule would be 144. Now if this were *correct,* our possible products would lie between 143.5 and 144.5. Upon multiplying 11.65 by 12.25 the least possible product is found to be 142.7125. Is this number within the limits of the product, 144? No, because it lacks .7875 of being equal to 143.5. The greatest possible product equals 11.75×12.35 or 145.1125 which is seen to be .6125 greater than 144.5. Surely in this case the product 144 cannot be said to be correct. Nevertheless, 144 is a *fair average value* for the product since it lies between the least possible product and the greatest possible product. Although this product may or may not be correct, its fair average value makes it a *reasonable* product for general measuring purposes. To distinguish this *reasonable* product from correct products place "approximately" directly following it, *viz.,* 144 sq. ft. approximately.

If this approximate product is not sufficiently informative for an especially exacting project, and more precise data are not available the least possible and the greatest possible products may always be determined and used as the situation demands. In this case the products should be labeled "Least possible product" and "Greatest possible product" *so that it may be known exactly to what the numerical expressions refer.* RULE: THE PRODUCT OF APPROXIMATE NUMBERS MAY CONTAIN NO MORE SIGNIFICANT FIGURES THAN THE LEAST ACCURATE FACTOR CONTAINS.

Exercises: (*Label the answers as suggested above.*)

1. Let the pupil show why it is reasonable to record the area of a plot 4.2 ft. long and 8.5 ft. wide by using only two significant figures.

2. Given $a=23.34$ $b=66.92$. *Find ab.*

3. Given $a=234$ $b=21$. *Find ab.*

4. Given $a=37.92$ $b=93.75$. *Find ab.*

5. Given $x=83600$ $y=213$. *Find xy.*

6. Find the circumference of a circle whose diameter measured to the nearest foot is 12.[13]

[13] Use of π in Multiplication and Division with Approximate Numbers. π is an approximate

7. What is the greatest possible area of a rectangular field whose dimensions are correctly given as 23.4×18.3 rods?

8. What is the least possible area of a square garden whose side is correctly expressed as 38 ft.?

9. Since areas are found by a process of multiplication, should the dimensions of plots be given with the same degree of precision? If a plot's length is given correctly as 286 ft., how precisely should its width be given if its width is less than 100 ft.?

Division of Approximate Numbers

In the division of two approximate numbers, as in multiplication, the quotient should contain no more significant figures than the number which contains the fewer significant figures.[14] (See also note 13.)

Exercises:

1. Divide 178.63 by 4.18.

2. The circumference of a circle is 25.8 ft. Find the diameter.

3. The volume of a room is 1230 ft. Its height is 15.2 ft. Find the area of the floor.

4. What current (amperes) is flowing through a wire whose resistance is 12 ohms, if the potential difference between the ends of the wire is 556.8 volts? Suggestion: The relationship between amperes, ohms and voltage is given by the equation: Amperes = voltage/ohms.

Addition of Approximate Numbers

a	b	
6.826	6.8	
7.52	7.5	
6.8	6.8	
.510	.5	
.087	.1	
21.743	21.7	Approximately

If the addends feature varying precision, as in a, they should be rounded to the same precision as the addend having least precision, as in b. The sum 21.7 is labeled "approximately" since it may or may not be correct. *It is the most reasonable sum that can be secured from the given addends.* This sum will serve for all general purposes.

If the above "approximate" sum is not sufficiently informative, the least possible and the greatest possible sums may be determined. Example: $21.4+3.5+81.6=106.5$ approximately. Since each addend may have a possible error of .05, the total possible error is $3\times.05=.15$. Thus, the least possible sum of the above addends is $106.5-.15=106.35$. Similarly, $106.5+.15=106.65$ equals the greatest possible sum.

Since equal precision among our addends is of first importance in determining our sum of measured numbers, we should always measure to the same unit of measure when the data are to be added.

Rule: The sum of approximate numbers may feature no greater precision than the least precise addend.

Exercises: *Add the following.* (*That is, find approximate sums.*)

1.		2.		3.	
	12.54		*6.5478*		*2*
	13.537		*2.00*		*3*
	2.376		*7.5*		*10*
			6.823		*125*

number because it is a rounded number. Since the figures dropped are known, π is slightly different from the more common approximate numbers. Thus, a measurement of 2.44 ft. may correctly refer to any length lying between 2.435 ft. and 2.445 ft. Its possible error is .005 of a foot. When 3.14 is used as a value of π the known error is a little less than .0016. Because of this comparatively small error in the rounded value of π the most accurate results may be obtained by using a value for π featuring one more figure than its partner approximate number.

Examples: When multiplying (or dividing) 21 by π use 3.14.

When multiplying (or dividing) 12.3 by π use 3.142.

When multiplying (or dividing) 245.6 by π use 3.1416.

[14] When expressing a quotient to two significant figures, it is necessary to carry the division process far enough to see what the *third* figure is going to be so that the quotient can be rounded reasonably to *two* places. Similarly, when desiring a quotient to be expressed with four significant figures the fifth figure should be ascertained so that the fourth figure may be changed as the policy of rounding off would indicate.

4. $\begin{array}{r} 5{,}000 \\ 1{,}200 \\ 1{,}700 \\ 35{,}000 \\ \underline{1{,}000} \end{array}$ *5.* $\begin{array}{r} 13.5 \\ 14.58 \\ \underline{9.0} \end{array}$ *6.* $\begin{array}{r} \$28. \\ 3.50 \\ \underline{6.33} \end{array}$

7. Find the least possible sum of 24.2+8.3+247.8.

8. Find the greatest possible sum of 24.23+0.46+24.11+0.08.

Subtraction of Approximate Numbers

As in addition, if the minuend and subtrahend do not feature the same precision, round off the more precise number so that it features the same precision as the least precise number. Here again the remainder must be labeled with "approximate" since it is merely a "reasonable" remainder. The least possible remainder and the greatest possible remainder should be considered when the "approximate" remainder is not acceptable.

Exercises: *Subtract the following and label as suggested.*

1. $\begin{array}{r} 15.9 \\ \underline{8.74} \end{array}$ *2.* $\begin{array}{r} 9.2 \\ \underline{8.0} \end{array}$ *3.* $\begin{array}{r} 67ooo \\ \underline{17oo} \end{array}$ *4.* $\begin{array}{r} 8.764 \\ \underline{2.1} \end{array}$

5. What is the least possible remainder of Ex. 1?

6. What is the greatest possible remainder of Ex. 2?

Extracting Square Root

In extracting the square root of an approximate number, the root may be carried to as many significant digits as appear in the original number, even though it is necessary to supply zeros. This may be verified by multiplication.

Exercises:

1. Extract square root of 196.4
2. Extract square root of 89.46
3. Extract square root of 167.42
4. Extract square root of 9827.45

Please Renew May Expirations Now!

Members whose subscriptions expired with the May number have received notices of the fact together with a request to renew their subscriptions at once. These notices were sent out early to forestall the competition of summer and vacation needs which sometimes cause renewals for The Mathematics Teacher to be sidetracked for several months.

It is a great service to the business manager to know *now* what funds he has to work with and how many subscribers he can count on for the coming year. Much of the work of editing and producing a professional journal has to be arranged for months in advance of the actual issues. It is extremely difficult, if not impossible, to make these necessary arrangements unless the support of our members is assured. It is for that reason that members are asked to send in their renewals and $2.00 fees promptly.

Since there are no issues of The Mathematics Teacher in June, July, August, and September, subscriptions which started with the October 1938 number close with the May 1939 number. Those who became members last fall, starting with the October number, are also urgently asked to make their renewals now for the coming year in the light of the considerations outlined above.

An Experimental Study of a New Mathematics Test for Grades 7, 8, and 9

By Margaret Seder
Educational Records Bureau, New York, N. Y.

With the increased emphasis on integrated mathematics courses in the junior high school, rather than the plan whereby arithmetic was taught in Grades 7 and 8 and algebra in Grade 9, teachers began to feel the need for an objective achievement test which contained some arithmetic, some simple algebra, and some elementary geometry. Such a test was not available at the junior high level, so plans were made for the construction of such a test by teachers of mathematics in schools belonging to the Educational Records Bureau in co-operation with the Cooperative Test Service of the American Council on Education.

The method followed was that of test construction by a committee[1] which was assisted by teachers from other Bureau schools in determining the objectives to be tested and in criticizing the test. The list of twelve objectives for general mathematics at the seventh, eighth, and ninth grade levels, agreed upon by various schools, together with the median per cent of class time devoted to achieving each objective is as follows:

Median per cent

17 *1. Development of the ability to interpret and manipulate simple types of relationships when expressed in algebraic notation (e.g., formulas and equations).*

11 *2. Development of the ability to recognize functional relationships in everyday experience and in simple problems; to identify the essential elements of these relationships; and to express simple types of these mathematically.*

6 *3. Development of an understanding of the mathematical symbols, definitions, and vocabulary necessary to understand quantitative statements in general reading, school subjects, and mathematical textbooks.*

6 *4. Development of the ability to interpret and construct graphical representations and to recognize situations in which graphical methods are appropriate.*

7 *5. Development of a sense of space relations including shape, size, and symmetry by constructing geometric figures and learning facts about them, and of an appreciation of geometric forms in nature, art, architecture, and industry.*

4 *6. Development of the ability to exercise judgment in handling approximate numbers in various measurement situations; development of the ability to estimate size, to measure and to compute size both directly and indirectly.*

25 *7. Development of skill in the use of the fundamental processes applied to both algebraic and arithmetic quantities including integers, fractions, and decimals.*

10 *8. Acquisition of the information necessary for the application of mathematics to financial problems such as those of insurance, taxation, banking, money and money management, simple interest, discount, commissions, and profits.*

3 *9. Development of an appreciation of the value of mathematics in the advancement of civilization.*

1 *10. Development of the ability to summarize certain characteristics of a group of measures. (Simple statistical concepts such as mean, median, range, etc.)*

6 *11. Development of the ability to understand and manipulate quantitative rela-*

[1] The committee was composed of Miss Alice H. Darnell, of the Germantown Friends School; Miss Rose E. Lutz, of the Radnor High School; Mr. Stevenson W. Fletcher, Jr., of the George School; and Dr. John C. Flanagan, of the Co-operative Test Service, chairman.

tions in the form of ratios and proportions.

3 *12. Development of the ability to predict results approximately.*

1 *13. Ten Bureau schools added an objective and allotted from 1 to 20 per cent of the instructional time to the objective. In most cases, it was found that the added objectives could be classified under one of the other twelve objectives.*

The various objectives seemed to fall into four general categories: (1) mathematical skills, (2) mathematical facts, terms and concepts, (3) mathematical applications, and (4) appreciation of the nature and value of mathematics. The committee then undertook to construct questions which would measure these objectives, and since the committee felt that most questions could be formulated in a multiple-choice form, this type of question was used. The wrong choices were made to conform to experience with typical errors made by pupils.

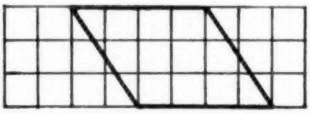
Two forms of the resulting Cooperative Mathematics Test for Grades 7, 8, and 9 are now available,[2] and the committee is constructing a third form which will be published in the spring of 1940. Each form includes 45 items on skills which are allotted 30 minutes, 30 items on facts, terms, and concepts which are allotted 10 minutes, 30 items on applications which are allotted 30 minutes, and 25 appreciation items requiring 10 minutes. The whole test, therefore, contains 130 items and requires 80 minutes of testing time.

The committee recognizes that the names of the four parts merely denote emphases and that overlapping is necessary. However, intercorrelations to be presented later are low enough to indicate that different aspects of mathematical achievement are being measured. These intercorrelations do not indicate what aspects are being tested, but a careful classification of items by the committee with criticisms and suggestions from the teachers was the method used to obtain a valid classification.

Typical items selected to represent the kind of items included in each section are presented below.

SKILLS

Fig. 3

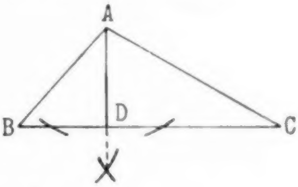


26. *If each square in Figure 3 above is 1 ft. long, the area of the parallelogram is*
    26-1 *18 sq. ft.*
    26-2 *16 sq. ft.*
    26-3 *12 sq. ft.*
    26-4 *24 sq. ft.*
    26-5 *6 sq. ft.* . . . . ( )

45. *In the equation* $\frac{2a}{5}-\frac{3}{4}=\frac{3a}{20}$, *the value of a is*
    45-1 -3
    45-2 $\frac{15}{11}$
    45-3 *3*
    45-4 *4*
    45-5 *5* . . . . ( )

FACTS, TERMS, AND CONCEPTS

Fig. 12



59. *AD in Figure 12 is*
    59-1 *a perpendicular bisector*
    59-2 *an angle bisector*
    59-3 *an hypotenuse*
    59-4 *an altitude*
    59-5 *a median* . . . . ( )

61. *An amount of money paid to a company for insurance against loss is a*
    61-1 *premium*
    61-2 *commission*
    61-3 *discount*
    61-4 *deposit*
    61-5 *tax* . . . . ( )

APPLICATIONS

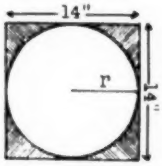
96. *Two planes start from St. Louis at the same time. One flies directly east at a speed of 110 miles per hour, and the other directly west at a speed of 140 miles per hour. If their radio intercommunication systems enable them to*

[2] Published by the Cooperative Test Service, 15 Amsterdam Avenue, New York, N. Y.

talk to each other at distances up to around 1000 miles, for about how long a period will it be possible for them to converse?
96-1 *$33\frac{1}{3}$ hr.*
96-2 *40 hr.*
96-3 *3 hr.*
96-4 *4 hr.*
96-5 *5 hr.* . . . . ( )

101. *A rough approximation of the area of the shaded portion of Figure 21 was needed in ordering some packing material. The shaded are is about*

Fig. 21



101-1 *100 sq. in.*
101-2 *150 sq. in.*
101-3 *10 sq. in.*
101-4 *40 sq. in.*
101-5 *5 sq. in.* . . . . . ( )

APPRECIATION

106. *The number 35 means*
106-1 *3+5*
106-2 *3 times 5*
106-3 *3 times 10+5*
106-4 *3 divided by 5*
106-5 *3 raised to the fifth power* . . . . ( )

120. *Astronomers are able to predict eclipses of the sun because of*
120-1 *the color of the sun*
120-2 *the location and distances of the stars*
120-3 *observation of the span of years between eclipses in the past*
120-4 *the temperature of the earth*
120-5 *accurate measurement of the paths of the moon and the earth* . . . . ( )

fact that the tests are eighty minutes in length rather than forty minutes as are all other Form O and P Cooperative mathematics tests. The tests were also rather widely used in public schools. Data from the independent schools were available for use in an objective evaluation of the test results. Evidence to be presented here concerns the reliability of the tests, the intercorrelations of subtests, and the relation of total scores to scores on other mathematics tests.

Information concerning the reliability of the subtotal and total scores at a single grade level for Form O and Form P of the test is presented in Table I. These coefficients were obtained by correlating scores on odd-numbered items with scores on even-numbered items and estimating the reliability of the whole by means of the Spearman-Brown formula. The internal consistency of the parts is quite satisfactory for group prediction, and for both forms, the reliability of the total score is high. The reliability of the appreciation section is lower in Form P than in Form O, but this is probably because in Form O the section contained several items on the placement of decimal points which

TABLE I

RELIABILITY COEFFICIENTS BASED ON THE PART AND TOTAL SCORES OF 154 EIGHTH GRADE PUPILS FOR FORM O AND 170 EIGHTH GRADE PUPILS FOR FORM P OF THE COOPERATIVE MATHEMATICS TEST FOR GRADES 7, 8, AND 9.*

Part	No. of Items	Time	Form O			Form P		
			r	Mean	S.D.	r	Mean	S.D.
Skills	45	30	.881	32.9	6.4	.879	24.9	6.3
Facts, Terms and Concepts	30	10	.687	17.9	4.3	.695	16.5	4.1
Applications	30	30	.811	20.8	4.8	.861	16.4	5.6
Appreciation	25	10	.811	13.1	4.2	.718	10.9	3.2
Total	130	80	.940	84.6	16.6	.924	68.7	16.1

* Coefficients are odd-even reliabilities predicted by the Spearman-Brown formula.

The need for the test and the success of the undertaking is indicated in part by the fact that thirty-seven member schools of the Educational Records Bureau used Form 0 of the Mathematics Test for Grades 7, 8, and 9 in one or more of these grades in 1938, while thirty schools used Form P of the test in 1939 in spite of the

were largely of a speed nature. It is felt by the committee that this lowering of reliability may be the result of increasing the validity of the appreciation section by omitting such items which seemed better classified as skills.

It is interesting to note at this point the correlation between Form O and Form P

when the tests are taken one year apart. These correlations are presented in Table II. Both of these correlations are quite

TABLE II

Correlations Between Form O and Form P of the Cooperative Mathematics Test for Grades 7, 8, and 9 Taken One Year Apart

Subjects	Number of Cases	Correlation
Grade VIII* pupils	90	.845
Grade IX* pupils	99	.902

* Grade placement at time of second test (Form P).

high, and for ninth grade pupils, the correlation between the forms is almost as high as the predicted reliability of either form for eighth grade pupils. These coefficients indicate that the aspects of mathematical achievement being measured are quite stable.

The alteration of the appreciation section in Form P has resulted in lowering its correlation with the skills section and with the facts, terms and concepts section, as is shown in Table III which contains

TABLE III

Intercorrelations of Subtests of the Cooperative Mathematics Test for Grades 7, 8, and 9 Based on the Scores of 154 Eighth Grade Pupils for Form O and 170 Eighth Grade Pupils for Form P

Parts	Form O	Form P
Skills with Facts, Terms, and Concepts	.565	.572
Skills with Applications	.707	.716
Skills with Appreciation	.547	.485
Facts, Terms, and Concepts with Applications	.588	.524
Facts, Terms, and Concepts with Appreciation	.646	.463
Applications with Appreciation	.559	.588

correlations among the subtests for Form O and Form P.

In both forms, skills and applications are more highly correlated than any other pair of subtests. For Form O, the next highest correlation is between facts, terms, and concepts and appreciation, while for Form P this pair of subtests shows the lowest intercorrelation. The relation of applications to appreciation is the second highest correlation for Form P. Although as would be expected, these various aspects of achievement in mathematics show rather substantial correlation, the fact that these coefficients are lower than are the reliability coefficients indicates that several relatively distinct aspects of mathematical achievement are being tapped.

All of this evidence points to the fact that in terms of reliability of a single form, correlation between forms, and relation among subtests, the Mathematics Test for Grades 7, 8, and 9 is a good test. Additional information concerning the value of the test may be gained from a consideration of the relation of scores on this test, to scores on other mathematics tests. For example, in Table IV are shown correlations between Form O of the mathematics test and Form P of the Cooperative Algebra Test taken one year apart.

The correlation between the mathematics test and intermediate algebra is somewhat lower than the correlation with elementary algebra. This is to be expected since the type of algebra contained in the mathematics test is similar to that in the elementary algebra test while it is rather different from much of that included in

TABLE IV

Correlations Between Form O of the Cooperative Mathematics Test for Grades 7, 8, and 9 and Form P of the Cooperative Algebra Tests Taken One Year Apart

Subjects	Number of Cases	Algebra Test	Correlation
Grade IX* pupils	150	Elementary	.783
Grade X* pupils	67	Intermediate	.591

* Grade placement at time of second test (Form P).

the intermediate test, which is devoted to quadratics and topics following quadratics. Even so, the correlation between the mathematics and intermediate tests is as high as is usually found between tests for different subjects in the same field. The correlation between mathematics and elementary algebra is high enough so that one can predict, at least within broad categories, such as "high," "average," and "low," the success of an individual on the Co-operative Elementary Algebra Test from his score on the Mathematics Test for Grades 7, 8, and 9 taken one year previously. This suggests that the test would be useful for determining admission to or placement in elementary algebra classes as well as for measuring achievement in general mathematics.

The correlation between the mathematics test and the elementary algebra test when both tests are taken at approximately the same time is also about +.8. This correlation is shown in Table V, which also includes correlations between Form P of the mathematics test and the two arithmetic tests of the Metropolitan Battery, Advanced Form B.

All correlations in this table are substantial. It is evident that the mathematics test correlates to the extent of about .8 with all of these tests. For both Grade VII and Grade VIII pupils, the correlation of the mathematics test with the Metropolitan problems test is slightly higher than the correlation with the fundamentals test. It is interesting to know in this connection that the correlation between fundamentals and problems, based on the scores of the seventh grade pupils is .759, and the correlation based on the scores of the eighth grade pupils is .763. These correlations are slightly higher than the correlations between skills and applications on the mathematics test. In general, the correlations in Table V indicate that any group which does well on Form P of the Mathematics Test for Grades 7, 8, and 9 is likely to do well also on the Metropolitan arithmetic subtests or on the Co-operative Elementary Algebra Test. The correlations are not high enough to predict an individual's score on one test from his score on the other, but they are high enough so that predictions within such broad groups as "good," "fair," and "poor," can be made with a satisfactory degree of accuracy.

TABLE V

Correlations Between Form P of the Cooperative Mathematics Test for Grades 7, 8, and 9 and Other Mathematics Tests Taken at Approximately the Same Time

Subjects	Number of Cases	Other Mathematics Tests	Correlation
Grade IX pupils	141	Co-operative Elementary Algebra, Form P	.807
Grade VII pupils	193	Metropolitan-Arithmetic Fundamentals	.725
		Arithmetic Problems	.760
		Fundamentals plus problems	.792
Grade VIII pupils	175	Metropolitan-Arithmetic Fundamentals	.764
		Arithmetic Problems	.835
		Fundamentals plus problems	.850

Summary

1. Scores on the subtests of the Co-operative Mathematics Test for Grades 7, 8, and 9 are fairly reliable and the total score is highly reliable.

2. The scores on Forms O and P of the test are highly related.

3. The subtests are interrelated to some extent but at the same time they are meas-

uring relatively distinct aspects of mathematical achievement.

4. Scores on Forms O and P of the mathematics test are quite closely related to scores on the Cooperative Elementary Algebra Test, both when the tests are taken at about the same time and when they are taken a year apart. The relationship is such that the mathematics test might well be used for admission to, or placement in elementary algebra classes. The correlation with the intermediate algebra test is somewhat lower but is still fairly high.

5. Scores on the mathematics test are also rather closely related to scores on the Metropolitan Arithmetic Fundamentals and Problems Tests.

All of the evidence presented here indicates that the Cooperative Mathematics Test for Grades 7, 8, and 9 is a satisfactory test and that it is measuring several aspects of mathematical achievement. The data also suggest that besides being useful for measuring achievement in general mathematics, the test may be used for admission to or placement in elementary algebra classes.

Mathematics in Progressive Education*

By Maurice L. Hartung
University of Chicago, Chicago, Illinois

The subject "Mathematics in Progressive Education" is a rather extensive one. Doubtless many of you think that there is nothing to be said on this subject because you believe that there *is* no mathematics in progressive education. Such a belief has little basis in fact, however. It is true, of course, that some of the oratorically gifted prophets of progressive education have aired their views to the effect that there is little place for mathematics in a progressive school. I can testify, on the other hand, that there are many so-called progressive schools in which quite traditional and fairly rigorous courses in mathematics constitute an important part of the curriculum. There is a good deal of confusion abroad in the land on this subject. Part of the confusion is due to the fact that many teachers of mathematics do not know what progressive education is. Probably the rest of it is due to the fact that many people who think they know the meaning of progressive education have very vague or even incorrect ideas concerning the nature of mathematics. I think it will be wise at the outset to attempt to clear up certain misconceptions about progressive education.

There are probably more people who have misconceptions of the meaning of progressive education than there are people who really understand it. Originally it was a movement of *protest* against the existing order in elementary education. The present progressive movement began with those who dared to rebel against the so-called "lockstep" in education. They sought to remove the emphasis from *subjects* of instruction, such as grammar, arithmetic, and geography, and focus it upon the *boys* and *girls*. They wanted to make the schools come into closer contact with the *interests* of children, to minister more directly to their *immediate needs*, to develop their personalities as well as their minds. They early recognized the range of individual differences, and they supported the Dalton plan, the Winnetka plan, the contract plan, and similar methods designed to cope with this problem.

Also present in the movement was an element of revolt against the prevailing psychology of learning. Instead of stressing specific habits, drill, and similar isolated psychological aspects, their approach was from the standpoint of the whole rather than the part. Hence they sought to develop the project method and became concerned with the problem of integration. Although the movement originally emphasized the doctrine of *interests* and the notion of *individual differences*, progressive educators are now putting more emphasis on learning which is functional or useful, not only from an individual point of view but also from a broader social point of view.

The movement took root in the elementary schools and only recently has been extended to the secondary schools. As boys and girls from progressive elementary schools began to enter the high schools, there was a demand for more progressive practices at the secondary school level. Progress in this direction was made but slowly, however. Perhaps this was because of the rigid college-entrance requirements which restricted the possibilities of curricular modification; or perhaps it was because the teachers were more subject-matter minded in the high schools than in the elementary schools and it was more difficult for them to make the child, rather than the subject, their center of interest. Perhaps it was because as the things to be learned became more complex, it was not so easy

* An address delivered at the meeting of the National Council of Teachers of Mathematics, Cleveland, Ohio, February 24, 1939.

to abandon the logical pattern of organization which exists in mathematics, the chronological organization of history, the traditional pattern of physics in which the subjects of mechanics, heat, sound, light, and electricity are still discussed in the textbooks in approximately the order in which they were scientifically developed.

Whatever the reasons may be, it is a fact that there are so-called "progressive schools" in which the *content* of the mathematics curriculum is quite the same as it is in thousands of other schools which are not considered progressive. These schools are progressive in the sense that they are trying to apply elements of progressive doctrine—for example, the teachers are sincerely interested in the pupils as *persons*—but not progressive in the sense that they are extensively modifying the curriculum, at least with respect to mathematics. The pupils are learning algebra and geometry as they are developed in the well-known textbooks, and the teachers are doing a good job of teaching. Laxity of standards, following of pupil "whims," sole dependence on projects, wholly individualized instruction—these are rarely found. On the contrary, standards are very high, but the spirit and the atmosphere of the schools are different. This is almost certainly due to the progressive point of view. The relationship between teachers and pupils is based upon a mutual respect for personality, and the subject-matter plays a role of secondary importance in the pupil-teacher relationship. Good teachers succeed in building such relationships with pupils whether they are in so-called progressive schools or not. But it is in terms such as these that one must seek to define the nature of progressive education.

Just as many people have misconceptions concerning progressive education, there are also many different ideas about the nature of mathematics and its possible contributions to a general education. There have been a number of attempts to define mathematics, but the one common characteristic of these definitions is that they are all unsatisfactory. Each stresses some partial aspect of the subject, like the blind men and the elephant, and one is left with a very distorted picture of it. One of the more interesting definitions which I have heard is the following: "Mathematics is what mathematicians do professionally." However imperfect this attempt at definition may be from some points of view, it at least is broad enough to avoid giving the distorted picture referred to above. I think it is futile to attempt a formal definition of mathematics. At the moment, I merely want to emphasize one point. I am sure that one cannot obtain a fair notion of the subject by an examination of some of the textbooks written for secondary school pupils. They also stress narrow aspects of the subject, and one can only obtain a broad concept of it after long study and perhaps some reading of the non-technical papers by experts in the field who have attempted to write semi-popular expositions of it. The word "mathematics" probably means different things to different people. It thus becomes quite possible for some to say there is no place for mathematics in progressive education, while others insist that there is much. For if by "mathematics" you mean some of the material found in some textbooks, then it is true that a progressive point of view rules much of it out. In fact, leaders of mathematical education have been urging the elimination of, or at least reduction of emphasis on, certain topics for years. But mathematics cannot be kept in the schools by a continual process of cutting down the existing curriculum—by eliminating topics, changing the order of topics or assigning them to different grade levels, or similar changes. What is needed is a broad and comprehensive view of mathematics as a science. People who have such views see the possibility of developing mathematics in quite different ways from those to which we have grown accustomed. Thus there have been certain developments in the mathematics curriculum brought about by teachers in progressive schools. The work of

Fawcett of the Ohio State University School on *The Nature of Proof* and the work of Boyce at Bronxville, New York, on *Socio-Economic Mathematics* are perhaps the most widely known examples.

There are many others working along more or less similar lines. Lack of time prevents any exposition of what these various people are doing. The point at the moment is that one of the commonest criticisms of much of the experimental work going on is that it is *not mathematics.* The remainder of what I have to say will be largely directed to a discussion of this issue.

It is essential to recognize that those who are developing new courses have at least two dominant purposes. They want their pupils to learn some mathematics, and this fact should not be overlooked. But the fundamental characteristic of progressive mathematics is that courses are so organized that pupils can come to understand how mathematics and the modes of thinking employed by mathematicians are useful in life. You may call this teaching for transfer, or anything you like. But it seems to me to be clear that if we are really to show the usefulness of our subject in life our courses are going to contain some experiences which have not been part of the ordinary mathematics curriculum in the past. The effort to show how mathematics is being used *today* inevitably brings into the course some things not found in most courses until very recently, and not found in very many even yet. But "life-situations" are here to stay, I believe, or else mathematics in the secondary school is not here to stay. We must bring our subject closer to life than we have been or we will continue to lose ground. Progressive teachers have taken the lead in emphasizing life-situations; or if you like, teachers who are stressing life-situations are in some degree progressive. It certainly does not follow, however, that the so-called life-situations being used are all appropriate. Some examples I have read seem quite artificial. It does not follow that we have developed appropriate methods of introducing and teaching life situations. Much remains to be done here. Finally, *it does not follow that introducing life-situations into essentially traditional courses is going to really solve the problems facing mathematical education.* I think we must undertake a really thorough reorganization of our content and our methods with respect to the mathematics itself.

Prior to the publication of the Thirteenth Yearbook of the National Council, by Fawcett on *The Nature of Proof*, one of the commonest misconceptions about his work was that it was all study of "life-situations." Reading the Yearbook should make clear to everyone that the *method of developing the geometry in the course* was also a very important factor. There is good authority for the method used.

E. H. Moore was one of the greatest mathematicians ever produced in this country, and he had a tremendous influence in making the United States outstanding from a mathematical point of view. His famous address as retiring president of the American Mathematical Society cannot be quoted too often. It was in 1902 that he said:

> To consider the subject of geometry in all briefness: With the understanding that proper emphasis is laid upon all the concrete sides of the subject, and that furthermore from the beginning exercises in informal deduction are introduced increasingly frequently, when it comes to the beginning of the more formal deductive geometry why should not the students be directed each for himself to set forth a body of geometric fundamental principles, on which he would proceed to erect his geometric edifice? This method would be thoroughly practical and at the same time thoroughly scientific. The various students would have different systems of axioms, and the discussions thus arising naturally would make clearer in the minds of all precisely what are the functions of the axioms in the theory of geometry. The students would omit very many of the axioms, which to them would go without saying. The teacher would do well not to undertake to make the system of axioms thoroughly complete in the abstract sense. "Sufficient unto the day is the precision thereof."

Fawcett deserves credit for having given this recommendation a thorough trial. The result is a different sort of course than we are used to, but from a mathematical

point of view, I think the course is richer, better—*a truer miniature of the science as a whole,* than is the ordinary course.

It seems to me that similar changes in method and content are necessary in other courses. What pattern of organization is to be followed in the course which will replace our traditional algebra? Clearly no one can give a definite answer to such a question. But the trend which is developing momentum stresses certain major concepts which are useful in solving problems of all sorts. Among these concepts are the following:

1. The concepts of *formulation* and *solution.* As one faces a problem rationally, his first step is to try to get it formulated in manageable terms. He must try to visualize the general nature of the solution, recognize the factors which may need careful study, define certain terms, make certain assumptions. To do these and related behaviors successfully the problem solver needs a clear notion of what it means to formulate a problem, and also what is meant by a solution.

2. The concept of *data.* In real problems as a rule, data of some sort are needed. At the same time one needs to know what is meant by such phrases as "adequate data," "relevant data." In the emerging mathematics curriculum *real data* collected by the pupils either directly or from secondary sources such as books, magazines, etc., are becoming more and more prevalent. Hence the concept of data and the relation of data to problem solving may become important basic notions.

3. The concept of *approximation.* In real problems involving real data, the notion of approximation becomes fundamental. The term may cover such simple matters as rounding off numbers and the handling of significant digits, and it may also be extended to cover the entire theory of statistics, for statistical methods are approximate methods. The emerging mathematics curriculum will almost certainly put considerable emphasis on statistical ideas, for it is in this field that mathematics is finding ever increasing uses.

4. The concept of *function.* When one has formulated his problem, and collected his data, he has as a rule established certain correspondences. Thus the data are often put in tabular or graphical form, and the concept of function becomes useful. The function concept has been suggested by some writers as the one best suited for unifying the mathematics curriculum. In the development being outlined, this concept is recognized as one of several very important unifying notions, but the job of integrating the mathematics curriculum is not unloaded upon the function concept alone.

5. The concept of *operation.* As part of the effort to obtain a solution, one must do certain things with the data. Numbers must be added or multiplied, equations must be solved, and many other *operations* performed. Most of elementary mathematics today is primarily operational. Hence the concept of operation cannot be ignored. But it is only one of a number of important ideas which are characteristic of mathematics.

6. The concept of *proof.* Although the approach to real problems may be more often inductive than deductive the time nevertheless often comes when one must ask: "Has this been proved?" The concept of proof, with the notion of the relation of conclusion to initial assumptions and data and related notions, is certainly one of the fundamental ideas in mathematical work.

7. The concept of *symbolism.* The problem of communication is largely one of arriving at an appropriate symbolism. For primitive ideas, a symbolism of gestures, grunts, and yells may suffice. But to communicate complex ideas with a minimum of ambiguity, one needs to understand the nature of symbolism. Mathematics makes much use of it, and thus symbolism may be taken as one of our fundamental notions.

8. The concept of *mathematics* as an *evolutionary development.* That mathematics has come into existence, grown, and changed in response to needs, practical and esthetic, of different people at differ-

ent times, is too often not recognized. One of the fundamental things about mathematics is its relation to the development of civilization. This relation is insufficiently exposed at present, but we may hope to see it better handled in the future.

The Progressive Education Association has a Commission on the Secondary School Curriculum. The Commission has a sub-committee on Mathematics. The eight fundamental concepts listed above have been chosen by that Committee as an anchorage to which teachers of mathematics might moor during the stormy period through which we are passing. It is hoped that the Report of the Committee will give guidance to teachers who are undertaking a fundamental reorganization of the curriculum. Such reorganization, if sound, must certainly center around fundamental notions. Are these suitable, or if not, what better ones can be suggested?

In closing, it may be worth-while to point out once more that when really outstanding scientists bother to think about curriculum problems, they think in such broad terms as have been indicated above. As teachers of mathematics we may not yet know how to work out the details of reorganized courses, but if the ideas of the big men are to be followed, then at least the general direction is clear. In partial substantiation of this claim, I may quote from an address by the English physicist Charles Galton Darwin on the occasion of his retirement last August from the presidency of the British Association for the Advancement of Science. The address was entitled: "Logic and Probability in Physics" and was published in the magazine *Science*. Among other things, Mr. Darwin had this to say:

> My next point is one on which I do very much hope that there may be a consensus of agreement. This is that the subject of probability ought to play an enormously greater part in our mathematical-physical education. I do not merely mean that every one should attend a course on the subject at the university, but that it should be made to permeate the whole of the mathematical and scientific teaching not only at the university but also at school. To the best of my recollection in my own education I first met the subject of probability at about the age of thirteen in connection with problems of drawing black and white balls out of bags, and my next encounter was not till the age of twenty-three, when I read a book—I think it was on the advice of Rutherford—on the kinetic theory of gases. Things are better now, but mathematicians are still so interested in the study of rigorous proof that all the emphasis goes against the study of probability.
>
> Its elements should be part of a general education also, as may be illustrated by an example. Every month the Ministry of Transport publishes a report giving the number of fatal road accidents. Whenever the number goes up there is an outcry against the motorists, and whenever down, of congratulation for the increased efficiency of the police. No journalist ever seems to consider what should be the natural fluctuations of this number. A statistician answers at once that the natural fluctuation will be the square root of the total number, and apart from obvious seasonal effects that is in fact about what the accidents show; the number is roughly 500 ± 25. The proof of this does not call for any difficult mathematics, neither the error function nor even Stirling's formula, but can be done completely by the simple use of the binomial theorem. There is no mathematical difficulty that should trouble a clever boy of 15; *it is only the train of thought that is unfamiliar, and it is just this unfamiliarity that is the fault of our education.* The ideas and processes connected with the inaccuracy of all physical quantities are much easier to understand than many ideas that a boy has to acquire in the course of his studies; it is only that at present they are not taught, and so when met they are found difficult.

It is clear that Mr. Darwin includes under the term *probability* much that we would be more likely to call *statistics*. But regardless of terminology, last summer the distinguished President of the British Association and most of the teachers who were attending the Summer Workshops of the Progressive Education Association were thinking along the same lines. To build a curriculum along those lines we must sentence ourselves to many years of hard labor. But it is not labor on the rock pile. It is labor upon a more glorious mathematical edifice in which the boys and girls of the next generation may enthusiastically dwell.

Report of the Twentieth Annual Meeting of the National Council of Teachers of Mathematics, Cleveland, Ohio

By Edith Woolsey, *Acting Secretary*

The twentieth annual meeting of the National Council of Teachers of Mathematics was held in Cleveland, February 24–25, 1939. Five hundred thirty-three registered. The general theme was: "Mathematics Which Functions." The program as published in The Mathematics Teacher for January, pages 30–32, was carried out with some alterations.

Discussion Banquet was held at 6:30 p.m. in the Rainbow Room. President H. C. Christofferson of the N.C.T.M. presided. 1. Dinner and discussion led by hosts at tables. 2. Music by Shaker Heights Junior High a Cappella Choir, directed by Mrs. Frances Willard Smith. 3. Greetings to Dr. E. R. Breslich and an expression of appreciation for his inestimable services to mathematics and mathematics teachers in anticipation of his retiring from teaching in the fall, by Dr. John P. Everett. 4. Response by Dr. Breslich. 5. Brief announcements. 6. Address: "Mathematics in Civilization," by F. R. Moulton, Secretary of the A.A.A.S., Smithsonian Institute, Washington, D.C.

The exhibit, prepared by the mathematics teachers of Cleveland and the neighboring cities, was unusually complete and progressive. Great credit is due to all who helped to prepare it.

Meetings of the Board of Directors

Meeting of the Board of Directors, February 24, at 9 a.m., Star Suite. Present: H. C. Christofferson, J. T. Johnson, Ruth Lane, Vera Sanford, W. S. Schlauch, E. R. Breslich, V. S. Mallory, Edith Woolsey, Kate Bell, Florence B. Miller, John P. Everett. Absent: Edwin W. Schreiber, W. D. Reeve, L. D. Haertter, William Betz, Martha Hildebrandt, M. L. Hartung, and R. R. Smith.

The meeting was called to order by President Christofferson, who read to those present the President's Report and agenda for the business meeting. It was moved by Miss Sanford and seconded by Mr. Schlauch that the minutes of the last meeting be accepted as published in The Mathematics Teacher. The motion was carried.

It was moved by Mr. Mallory, seconded by Mrs. Miller, and carried that we ask the Board of Directors to confirm our action by mail on the following issues:

1. Whether we should carry on our work if our income is decreased.
2. Whether to substitute a discussion dinner for a discussion luncheon.
3. Whether to adopt double morning sessions.
4. To appoint a committee to co-operate with the English Council with M. L. Hartung as chairman.
5. To publish the Joint Commission's Report as the Fifteenth Yearbook.
6. To go ahead with the summer meeting with the N.E.A. in San Francisco.
7. To reduce the price of the Fifteenth Yearbook (only) to $1.25 from $1.75.
8. To approve exhibiting mathematical materials but not textbooks at meetings of the National Council.

President Christofferson announced that M. L. Hartung had been appointed to represent the Council on The National Commission for Co-operative Curriculum Planning (4 above) which met in Detroit, February 20–21. The President announced that plans are underway for the summer meeting in San Francisco. Copies of the Treasurer's report were passed out by Miss Woolsey, Acting Secretary.

This was followed by a discussion of the

expenses of the President's office, the expenses of the Secretary-Treasurer's office and the ballot expense. Due to a question about the interpretation of certain items, it was moved by Mr. Schlauch, seconded by Miss Sanford, and carried that the Treasurer's report be submitted to the auditing committee for investigation and study. The President appointed Mr. Schlauch as the auditing committee. A report will be sent out later by mail.

Miss Ruth Lane gave her report on the work with affiliated clubs. There are now fifty-one affiliated organizations, thirteen of which have affiliated during 1938–1939. It was moved by Mr. Schlauch, seconded by Mr. Johnson, and carried that we accept Miss Lane's report. Miss Lane urged the formation of a strong Speaker's Service Bureau to help affiliated clubs to get speakers at a low cost. She recommended that all Council officers and directors report to her when they are going to speak on any program so she can pass the word along and help other groups to get the same speakers.

Mrs. Miller gave her report on the work with state representatives. She recommended that a membership card be sent to each member of the Council, in order to stress membership in the organization as well as the subscription to THE MATHEMATICS TEACHER. It was moved by Mr. Schlauch and seconded by Mrs. Miller that a membership card be worked out by Mr. Reeve and the editorial staff. Miss Sanford amended the motion to the effect that the expense be borne by the Council. The motion was carried. It was moved by Miss Sanford, seconded by Mr. Mallory, and carried that Mrs. Miller's report be accepted.

Mr. Christofferson appointed Mr. Schlauch chairman of the Budget and Finance Committee. It was recommended that the publication of a register of members be postponed for another year. It was moved by Mr. Schlauch, seconded by Mr. Breslich, and passed that the work of the Pamphlet Committee be accepted and the committee be discharged with thanks.

It was moved by Mr. Schlauch, seconded by Mr. Johnson, and carried that Mr. Schorling's committee be continued and that the chairman submit a report when ready together with an estimate of the expense. It was moved by Mr. Schlauch, seconded by Mr. Johnson, and carried that the President be empowered to expand the Visual Aids Committee as he sees fit. Mr. Christofferson then appointed Miss Kate Bell and Mr. E. R. Breslich as additional members of that committee. Members previously appointed are Mr. E. W. Schreiber and Miss Edith Woolsey.

New Business: The subject of a registration fee was discussed at some length. It was moved by Miss Bell, seconded by Mr. Breslich, and carried that we charge a registration fee of fifty cents for nonmembers for attendance at any one of the three meetings and that members be admitted by membership card.

The question of three meetings a year was discussed: should they be adopted permanently or not? It was moved by Mr. Schlauch, seconded by Mr. Mallory, and carried that we adopt the general policy of a summer meeting and a December meeting, in addition to the annual February meeting, unless otherwise voted by the Board or unless duplication of areas makes it inadvisable.

Mr. Christofferson suggested that we have a program committee to function for three years. This was followed by a discussion but action was postponed until the evening meeting.

It was moved by Miss Sanford, seconded by Mr. Schlauch, and carried that we give a vote of appreciation for Mr. Reeve's excellent work and enthusiastic support of the Council. Mr. Schlauch reported on a suggestion of Mr. Reeve's that we increase the Editorial Board to four members. It was found that the bylaws of the Council provide for an editor and two associate editors. Therefore, it would be impossible at this time to in-

crease the number of associate editors. It was suggested that Mr. Reeve organize a number of departments and make more people responsible. It was moved by Mr. Schlauch, seconded by Miss Sanford and carried, that the President be empowered to appoint an Editorial Committee with power to act to renew the contract with Mr. Reeve for publication of the magazine and yearbooks and to suggest such modifications as seem advisable in the organization of assistants. Mr. Christofferson appointed the following Editorial Committee: Mr. Breslich, chairman; Mr. Schlauch; Miss Sanford. Mr. Christofferson suggested that we might have a department in the magazine devoted to questions and problems, and another one on new books and equipment.

It was moved by Mr. Schlauch, seconded by Mr. Mallory and Mr. Johnson, and carried that we authorize Mr. Reeve to grant permission to Mr. Bakst to reprint any portion of the Twelfth Yearbook when the present edition has been sold. The question of publishing a second monograph was discussed. It was moved by Mr. Mallory, seconded by Mrs. Miller, and carried that the Editorial Committee be authorized to talk with Mr. Reeve about this monograph and approve its publication.

It was moved by Miss Sanford and seconded unanimously that we send messages to Mr. Reeve, Mr. Betz, Miss Hildebrandt, and Mr. Schreiber expressing our regrets that they were not present. At 12:30 the meeting adjourned until after the evening meeting.

Meeting of the Board of Directors, 10:30 P.M., February 24, Star Suite. Present: H. C. Christofferson, J. T. Johnson, Ruth Lane, Vera Sanford, W. S. Schlauch, E. R. Breslich, V. S. Mallory, Edith Woolsey, Kate Bell. Absent: E. W. Schreiber, W. D. Reeve, L. D. Haertter, William Betz, Martha Hildebrandt, M. L. Hartung, R. R. Smith, Florence Brooks Miller.

The President announced he would communicate our decision on Mr. Bakst's case to Mr. Reeve.

Mr. Mallory reported on the work of the committee on which he and Mr. Hartung have been serving relative to our having a permanent paid executive secretary. They suggest that the President hire someone for part time, probably for $50 a month. This position might develop into a full time job when the size of the Council warrants it. It was moved by Mr. Schlauch, seconded by Miss Sanford and Mr. Mallory, and carried that the Council authorize the President and Mr. Reeve to engage, if they think it advisable, an executive secretary at a cost not to exceed $600 a year to take over such duties as they may determine.

Mr. Breslich moved that it be the policy of the Council to request that each one to whom funds are budgeted and appropriated shall render an itemized statement of the use of the money to the auditor appointed by the Board. The motion was seconded by Mr. Mallory and carried.

It was moved by Miss Bell, seconded by Mr. Breslich, and carried that we authorize the President to appoint a program committee according to his plan, as follows: A committee having three subcommittees, to work in the following fields: 1. Mathematics in the elementary school, 2. Mathematics in the secondary school, 3. Teacher training; this committee is to have charge of all programs of the Council in the areas indicated for each convention. This will foster a long view plan for each area. Each sub-committee is to be composed of at least three members with overlapping terms so that each would serve for at least three years and not over one-third would be new each year. No one can succeed himself as a member of any of these committees. The meeting was adjourned.

Annual Business Meeting of the National Council of Teachers of Mathematics

February 25, 4:00 P.M. Coral Room,

Carter Hotel. Attendance 50. The meeting was called to order by President Christofferson. The secretary reported the results of the election as follows: Second Vice President, E. R. Breslich; Members of Board of Directors, Virgil S. Mallory, A. Brown Miller, Dorothy Wheeler.

President Christofferson gave a brief report of the business which the Board had transacted at its two meetings on Friday. He also asked the members to co-operate with him in making the meetings better by sending in suggestions for future meetings. It was moved by Mr. Allan R. Congdon, seconded by Mr. J. H. Hlavaty, and carried that the Council send messages to the seven absent Board members expressing our appreciation for their past services and our regrets at their inability to be at this meeting. Mr. Congdon and Mr. Hlavaty were appointed as a committee to compose this message. Dr. W. W. Rankin of Duke University suggested that the group express its appreciation to all who have worked to make this convention a success. He also expressed the desire for closer contacts between secondary and college mathematics teachers. Dr. F. L. Wren of Peabody College moved that we express our appreciation to the Cleveland Mathematics Club for this excellent meeting. It was seconded by many and passed unanimously with a rising vote. It was moved by Miss Voorhees, seconded by Miss Dornell, and passed that we extend our appreciation to the Carter Hotel for the excellent accommodations and services they gave us. The meeting adjourned.

Meeting of Board of Directors, February 25, at 5:00 P.M. Coral Room, Carter Hotel. Present: H. C. Christofferson, A. Brown Miller, W. S. Schlauch, Kate Bell, Dorothy Wheeler, Ruth Lane, E. R. Breslich, V. S. Mallory, Vera Sanford, Edith Woolsey. Absent: William Betz, W. D. Reeve, E. W. Schreiber, Martha Hildebrandt, M. L. Hartung, R. R. Smith. President Christofferson welcomed the new officers and directors.

There was an informal discussion on ways of arranging programs to increase the interest of supervisors in the subject and the interest of mathematics teachers in the Council. The question of the value of having undergraduates as junior members of the Council was raised and also the feasibility of giving them a special rate. It was moved by Miss Bell, seconded by Miss Wheeler, and carried that the President appoint a committee to lay out a plan for organizing undergraduate mathematics clubs which may be affiliated with the National Council and have power to act. President Christofferson appointed the following committee: V. S. Mallory, chairman; Ruth Lane, E. H. C. Hildebrandt. The meeting adjourned.

MEMBERSHIP BY STATES IN THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS, APRIL 1939

Compiled by EDWIN W. SCHREIBER, *Secretary*

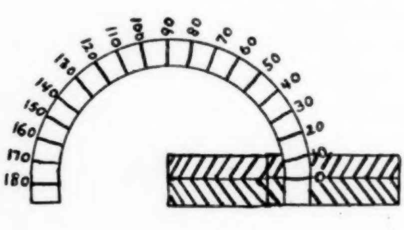
State	*April 1939*						
Ala.	57	Ind.	145	Neb.	79	S. C.	76
Ariz.	18	Iowa	121	Nev.	6	S. D.	18
Ark.	35	Kan.	161	N. H.	25	Tenn.	73
Calif.	210	Ky.	65	N. J.	221	Tex.	169
Colo.	74	La.	43	N. M.	15	Utah	12
Conn.	81	Me.	28	N. Y.	602	Vt.	25
Del.	16	Md.	66	N. C.	81	Va.	78
D. C.	62	Mass.	236	N. D.	24	Wash.	74
Fla.	50	Mich.	232	Ohio	290	W. Va.	48
Ga.	50	Minn.	117	Okla.	104	Wis.	134
Idaho	9	Miss.	59	Ore.	40	Wyo.	15
Ill.	482	Mo.	114	Pa.	358	Foreign	261
		Mont.	21	R. I.	29	Total	5409

Objective Materials in Junior High School Mathematics

By Joy Mahachek
State Teachers College, Indiana, Pennsylvania

In order to foster the use of objective materials, the course in Teaching of Secondary Mathematics at the State Teachers College, Indiana, Pennsylvania, each year includes a unit on the topic. The students, either individually or in groups, make objective materials which will contribute to a unit of work they are planning to teach. Perhaps the descriptions of some of these materials used in the unit on measurement will stimulate others to describe better materials and so help to build up a library of ideas for prospective teachers.

Angle measurement was illustrated by two strips of wood hinged together. A half circle marked off like a protractor was fastened at one end to one piece of the wood. The other end of the wooden half circle crossed the hinge and passed easily through a hole in the second piece of wood. With the instrument the idea of an angle as the amount of rotation of a line from its original position is most convincingly developed. Concepts of acute, right, obtuse and straight angles are also made plain. (Figure 1).



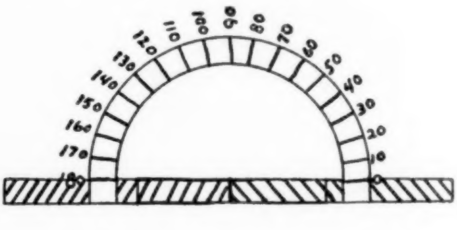


Fig. 1

Areas of geometric figures were developed through figures made of plywood with parts pegged together. The area of a parallelogram is easily developed by taking a detachable triangle from one side and pegging it into the other to form a rectangle, the formula for whose area is already known. Later another pegged parallelogram will separate into two equal triangles whose areas may be compared with that of the familiar parallelogram.

The relation between the diameter and circumference of a circle was demonstrated by a wooden circle cut along the diameter with the two equal parts hinged at the circumference. A piece of tapeline with large figures was tacked around the circumference and a second piece was tacked along the diameter. (Figure 2) The ratio of the circumference to the diameter in that particular circle was quickly found. Junior high school pupils were eager to make circles of other sizes to test the ratio.

The story of measures was developed through an interesting series of photographs inspired by the Ford pictures of early measures. One photograph showed the tallest boy standing behind the shortest girl as each measured the distance from the tip of the nose to the end of the

finger with a yardstick. Needless to say the hand of the youth six feet four inches tall extended beyond the yardstick while the fingers of the short girl could not reach the end of the stick. In a second picture a far. A third photograph of a foot rule and a shoe against a white background showed a surprising similarity in length. A fourth picture showed the variation in the width of hands as compared with four inches.

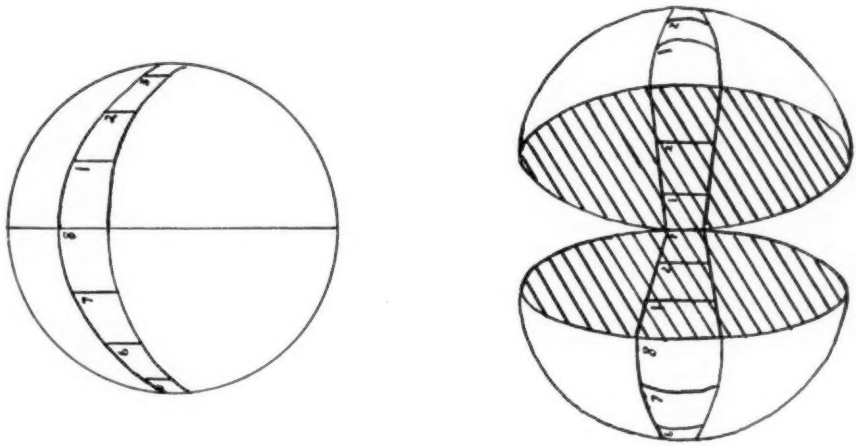


FIG. 2

fathom was marked off on the board in a heavy white chalk line. A tall boy and a short one were photographed before it with outstretched arms. Again the arms of the one extended beyond while the fingers of the other could not stretch so (Figure 3)

The students are always agreed that a unit takes on added meaning when they construct some piece of objective material and that therein lies a rich field for satisfying work in elementary mathematics.

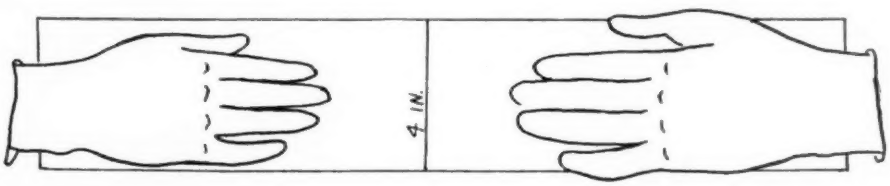


FIG. 3

Have You Paid Your Dues?

THE NOVEMBER issue of THE MATHEMATICS TEACHER will be sent *only to those whose dues are paid.* The date of expiration of your subscription is stamped on the wrapper of your October issue.

EDITORIALS

Mere Membership is Not Enough

It seems quite obvious that many members of the National Council of Teachers of Mathematics feel that they have done their duty when they have paid their dues which, of course, entitles them to one year's subscription to The Mathematics Teacher. However, we feel sure that if our members understood how much we need their help in securing new members and in holding old ones for continued membership, they would exert themselves a little more vigorously in campaigns for membership in the Council.

Examples have come to our notice this summer of teachers who are at work in rather prominent schools who have not even heard that such an organization as the National Council of Teachers of Mathematics exists. To be sure, we have a State Representative in each state of the Union, and these Representatives are doing a fine service for the Council. It is clear, however, that we cannot expect our Representatives to do all the work that is necessary to get new teachers properly enrolled in our organization and to hold some of the old ones in line.

Word comes to us that important meetings of mathematics teachers are held and no one takes it upon himself to say a word for the work of the Council. The yearbooks are not discussed and The Mathematics Teacher, the official journal, is not even mentioned.

Wouldn't it be a good idea for each and every member to consider himself a committee of one to help to put the work of the Council before the mathematics teachers of this country, regardless of the fact that certain other teachers are officially designated to represent us? We believe that it is only in this way that we can obtain a unified and enthusiastic group who will work for the best interests of mathematics in this country. The Mathematics Teacher will be glad to send to any individual who is interested in helping us secure new subscriptions application blanks describing the work of the Council which can be distributed at any meeting where mathematics teachers are gathered together. Moreover, we shall appreciate greatly any such co-operation that our members are inclined to give.

W.D.R.

The Fourteenth Yearbook

We are now ready to announce the forthcoming appearance of the Fourteenth Yearbook. The book was prepared by Mr. Ivan S. Turner and covers the training of mathematics teachers in the United States and England and Wales. It is to a large extent a comparative study and one in which all teachers of mathematics in each of these countries are sure to be interested. After all our problems at heart are very much the same. Moreover it is becoming increasingly clear that one of the most necessary improvements in connection with the keeping of mathematics on a high plane in this country involves the better training of teachers to work in the schools.

In this study, Mr. Turner prepares a set of guiding principles to be followed and then goes on to attempt to analyze what kind of success is being met in the three countries concerned in these matters. All teachers of mathematics will be interested in this new yearbook. As is the case with the former yearbooks, this new one can be secured from the Bureau of Publications, Teachers College, Columbia University, 525 West 120th Street, New York City. The price is $1.75, postpaid. See the complete advertisement on the back cover of this issue.

W.D.R.

Christmas Meeting of the National Council

The Christmas meeting of the National Council of Teachers of Mathematics will be held in Columbus, Ohio during the Christmas holidays. See the November's issue fcr further details and the program.

IN OTHER PERIODICALS

By NATHAN LAZAR
The Bronx High School of Science, New York City

1. Bennett, A. A., "The College Teacher of Mathematics Looks at Teacher Training." *The American Mathematical Monthly*, 46: 213–224. April, 1939.

An unusually frank discussion of a delicate problem. In Part I, the author makes some very pungent and true comments on the following topics: (a) the state of efficiency of the present secondary school teacher of mathematics; (b) the curriculum of colleges of education; (c) the offerings of the college which contribute toward preparation of secondary school teachers; (d) the prerequisites as to teaching performance for appointment to a college faculty; (e) the supervision to be exercised over the inexperienced young graduate assistant in his first year or two of instructional practice.

In Part II, the author discusses (a) the selection of prospective teachers, and (b) the object of examinations. In connection with the first topic the author draws incisive, almost pitiless, word-pictures of the following three types of teachers: (x) the professor as autocrat; (y) the absent-minded professor; and (z) the tutorial drudge. Readers of these descriptions will undoubtedly recognize their former instructors or their present colleagues.

The temptation is strong to quote phrases, sentences, and even whole paragraphs from this article. It will, however, be resisted, if for no other reason than to encourage the reader of this inadequate review to read the article in its entirety. Such scathing denunciations and true evaluations are rare, alas too rare, in our current pedagogical literature.

A bibliography is appended containing about forty-five items.

2. Black, M., "The Place of Logic in the Teaching of Geometry." *The Mathematical Gazette*, 23: 39–48. February, 1939.

A discussion of those phases of geometry which militate *against* the possibility of teaching it for transfer of skill in reasoning. "When we come to consider the techniques which will best promote the transfer of mathematical skill to non-mathematical situation, there will be one all-important guiding principle to consider. If we want transfer to occur we must see that ample exercise is given in the application of mathematical principles to the kind of subject matter to which we wish transfer to be made."

3. Blank, Laura, "The Solution of Problems by Means of Graphs." *School Science and Mathematics*, 39: 405–407. May, 1939.

An argument with examples for the desirability of teaching in the ninth year the solution of verbal problems by means of graphs.

The writer admits that "such problems as one would plot can be solved arithmetically, by algebraic equation, or by trial and error, perhaps more quickly than by means of a graph. Our reason for developing a graphical solution is that our method is an avenue of approach to the study of the formula and the linear system.'

4. Butler, Charles H., "The Indirect Method in Geometry." *School Science and Mathematics*, 39: 325–336. April, 1939.

The importance of indirect reasoning in geometry and in life is pointed out, and illustrations from both fields are worked out in great detail. The analysis follows closely that of Professor C. B. Upton and Professor H. C. Christofferson.

5. Charosh, Mannis, "On the Equation $x^2+y^2=z^2$." *The American Mathematical Monthly*, 46: 228–229. April, 1939.

The author offers a new solution of the above quadratic Diophantine equation. "It has the advantage of being related to the simple theory of the quadratic equation and is, therefore, suitable for presentation to a bright high school class in intermediate algebra."

6. Dobbs, W. J., "A Chapter on Commercial Arithmetic." *The Mathematical Gazette*, 23: 135–137. May, 1939.

An illuminating discussion of the questions that arise in the solution of problems dealing with instalment buying.

7. Ferrar, W. L., "Algebra in the Higher School Certificate." *The Mathematical Gazette*, 23: 144–149. May, 1939.

The author believes that the place accorded

in algebra to limits and convergence to be far too prominent, and the timing of the introduction of the more abstract notions of convergence to be premature. He regrets that the rigor of reasoning taught in geometry is not retained in the proofs of the theorems of algebra.

8. Harper, J. P., "Approximate Square and Cube Roots." *School Science and Mathematics*, 39: 316–319. April, 1939.

The writer presents methods for finding the square root or cube root of a number that are shorter than the familiar methods and are usually correct to four places.

"This method would seem to be highly commendable for use in connection with high school algebra where a knowledge of logarithms may be lacking. It is easy to learn and is easier to remember than the mathematical method."

Algebraic proofs of the methods are included.

9. Lankford, Francis G., Jr., "A Study of the Elements and Proofs of Plane Geometry." *Virginia Journal of Education*, 32: 126–128. December, 1938.

The author made a collection of items of plane geometry from selected sources, and by the application of a statistical technique he assigned a numerical value to each of them indicating its relative importance for a course in high school plane geometry. No effort was made to discover a list of items which are to be considered fundamental to such a course. The findings are intended to provide geometry teachers with a ranked list of items from which they may select content for their courses.

The detailed procedure for arriving at the ranked list consists of seven steps, and is described in great detail. The eighteen sources that were used fall into five categories: "Syllabi resulting from group or committee deliberations, syllabi resulting from individual authoritative opinion, syllabi weighing the items on the basis of use in theorems of a specified list, syllabi appearing in selected city and state courses of study, and modern textbooks."

The article concludes with a list of forty theorems and the relative value in per cent given to each of them.

In the space at our disposal it is impossible to evaluate the statistical technique, and the significance of the results obtained. This is especially difficult since the concepts "relative importance" and "fundamental to such a course" are not defined or explained. This study is, however, another welcome sign that all over the country research students are working on the problems of decreasing the number of theorems in plane geometry.

10. Olds, Edwin G., "Why Learn Mathematics?" *National Mathematics Magazine*, 13: 329–335. April, 1939.

One of the finest discussions on a difficult question. The writer realizes that it is futile for any one person to attempt a comprehensive and definite answer to so broad a question. He, therefore, proceeds in true mathematical fashion to analyze the *meaning* of the question before giving an answer to it. A bibliography of about thirty-five items is included, consisting of articles that appeared in The Mathematics Teacher, *The American Mathematical Monthly*, *The National Mathematics Magazine*, the yearbooks of the National Council of Teachers of Mathematics, and the Preliminary Report by the Joint Commission on *The Place of Mathematics in Secondary Education*.

11. O'Toole, A. L., "The Nature of Mathematics." *National Mathematics Magazine*, 13: 323–328. April, 1939.

Interesting remarks on what mathematics is not and what it is. A bibliography of fifteen items is included.

12. Sanders, S. T., "On the Applications of Mathematics." *National Mathematics Magazine*, 13: 304. April, 1939.

"As mathematical research becomes more highly specialized, we must expect the research mathematician more and more to leave to others the task of finding the applications of his research—if haply any should exist." It is, therefore, urged that a commission should be created to search for possible applications of mathematical techniques; otherwise the uses of mathematics will remain, as in the past, more or less unordered or subject to mere accidental discovery.

13. Stamp, Lord, "Education and the Statistical Method in Business—with Special Reference to Railway Statistics." *The Mathematical Gazette*, 23: 122–134. May, 1939.

An address by the chairman of the London, Midland, and Scottish Railway. In it are enumerated "thirteen particular uses of ordinary statistics applied in ordinary business in which quite good arithmetical students and others who have taken credits in School Certificate examinations or even higher, do not feel their way about very quickly." It is also pointed out that even the scholar who knows the theoretical phase of statistics is often faced with the difficulty of applying his knowledge to practical situations. The address concludes with an interesting story taken from railroad history illustrating the importance of finding a *unit* that can be handled easily.

14. Ullsvik, B. R., "New Materials and Equipment in the Teaching of Mathematics." *School Science and Mathematics*, 39: 432–442. May, 1939.

An interesting list of publications, materials, and equipment arranged under the following categories: (a) arithmetic, (b) publications in periodicals—administrative, philosophical, research, curricular, pedagogical, (c) supplementary available motion pictures, (d) literature on motion pictures, (e) posters, (f) puzzles, (g) equipment, (h) evaluation, instruments, (i) records, (j) commission reports.

A Code of Ethics for the Teaching Profession

Developed and published by National Education Association, 1201 Sixteenth Street, N. W., Washington, D. C., from whom it may be obtained in leaflet form.—Ed.

In order that the aims of education may be realized more fully, that the welfare of the teaching profession may be promoted, that teachers may know what is considered proper procedure, and may bring to their professional relations high standards of conduct, the National Education Association of the United States has developed this code of ethics.

Relations with Pupils and to the Community

The schoolroom is not the proper theatre for religious, political, or personal propaganda. The teacher should exercise his full rights as a citizen but he should avoid controversies which may tend to decrease his value as a teacher.

The teacher should not permit his educational work to be used for partisan politics, personal gain, or selfish propaganda of any kind.

In instructional, administrative, and other relations with pupils, the teacher should be impartial, just, and professional. The teacher should consider the different interests, aptitudes, abilities, and social environments of pupils.

The professional relations of the teacher with his pupils demand the same scrupulous guarding of confidential and official information as is observed by members of other long-established professions.

The teacher should seek to establish friendly and intelligent cooperation between the home and the school.

The teacher should not tutor pupils of his classes for pay.

Relations to the Profession and to Fellow Workers

Members of the teaching profession should dignify their calling in every way. The teacher should encourage the ablest to enter it, and discourage from entering those who are merely using the teaching profession as a steppingstone to some other vocation.

The teacher should maintain his efficiency and teaching skill by study, and by contact with local state, and national educational organizations.

A teacher's own life should show that education does ennoble.

While not limiting his services by reason of small salary, the teacher should insist upon a salary scale suitable to his place in society.

The teacher should not exploit his school or himself by personally inspired press notices or advertisements, or by other unprofessional means, and should avoid innuendo and criticism particularly of successors or predecessors.

The teacher should not apply for another position for the sole purpose of forcing an increase in salary in his present position.

School officials should not pursue a policy of refusing to give deserved salary increases to their employees until offers from other school systems have forced them to do so.

The teacher should not act as an agent, or accept a commission, royalty, or other reward, for books or supplies in the selection or purchase of which he can influence or exercise the right of decision; nor should he accept a commission or other compensation for helping another teacher to secure a position.

A teacher should avoid unfavorable criticism of other teachers except such as is formally presented to a school official in the interests of the school. It is also unprofessional to fail to report to duly constituted authority any matters which involve the best interests of the school.

A teacher should not interfere between another teacher and a pupil in matters such as discipline or marking.

There should be cooperation between administrators and classroom teachers, founded upon sympathy for each other's point of view and recognition of the administrator's right to leadership and the teacher's right to self-expression. Both teachers and administrators should observe professional courtesy by transacting official business with the properly designated person next in rank.

The teacher should not apply for a specific position unless a vacancy exists. Unless the rules of the school otherwise prescribe, he should apply for a teaching position to the chief executive. He should not knowingly underbid a rival in order to secure a position; neither should he knowingly underbid a salary schedule.

Qualification should be the sole determining factor in appointment and promotion. School officials should encourage and carefully nurture the professional growth of worthy teachers by recommending promotion, either in their own school or in other schools. For school officials to fail to recommend a worthy teacher for another position because they do not desire to lose his services is unethical.

Testimonials regarding a teacher should be frank, candid, and confidential.

A contract, once signed, should be faithfully adhered to until it is dissolved by mutual consent. In case of emergency, the thoughtful consideration which business sanction demands should be given by both parties to the contract.

Due notification should be given by school officials and teachers in case a change in position is to be made.

NEWS NOTES

Members of the National Council of Teachers of Mathematics met in a discussion luncheon July 5 in the International House of the University of California at Berkeley.

At each of the twenty-one tables was a discussion leader. Following is a list of topics discussed and the leaders.

"Recreational Values in Mathematics," L. J. Adams, Santa Monica Junior College, Santa Monica, California.

"Psychology and Mathematics Teaching," Reginald Bell, Associate Professor of Educational Psychology, Stanford University, California.

"Is Mathematics a Social Study?" George C. Bliss, Principal, Elizabeth Sherman School, Oakland, California.

"Reorganization of 10th, 11th, and 12th Year College Preparatory Courses for Those Who Have Had 9th Grade General Mathematics," William W. Booth, Claremont Junior-Senior High School, Claremont, California.

"High School Mathematics for the Non-College Group," Henrietta Burr, Ventura Union Junior High School, Ventura, California.

"A Reorganization of Academic Mathematics in Senior High School," Homer G. Cain, Pomona High School and Junior College, Pomona, California.

"Relational Thinking in Secondary School Mathematics," H. C. Christofferson, Professor of Mathematics, Miami University, Oxford, Ohio.

"Problems of the Mathematics Teacher in the Continuation High School," C. S. Cramer, McKinley Continuation High School, Berkeley, California.

"Meaning in Arithmetic," Gertrude M. Cross, Edwin Markham School, Oakland, California.

"Non-technical Mathematics for Graduating Seniors to Take with Them to a Work-a-day World," C. O. Eddington, South Gate High School, South Gate, California.

"Arithmetic in Senior High School," Bertha Fitzell, Eureka High School, Eureka, California.

"Applications in Geometry with Minimum Requirements," J. Calvin Funk, Santa Maria High School and Junior College, Santa Maria, California.

"Functional Thinking," F. L. Griffin, Professor of Mathematics, Reed College, Portland, Oregon.

"Measurement of the Ability to Use Quantitative Concepts in the Elementary Schools," Charles Grover, Principal, Glenview Elementary School, Oakland, California.

"How Can We Meet the Needs of the Junior College Student Who Has Had Little or No Mathematics in High School," Vern James, Menlo Junior College, Menlo, California.

"Enhancing the Content of Secondary Mathematics," Sophia Levy, Professor of Mathematics, University of California, Berkeley, California.

"Human Values in Mathematics," Frank Lindsay, Assistant Chief of Division of Secondary Education, Sacramento, California.

"One Mathematics Course for All in Ninth Grade But Given in X, Y, Z Sections," Earl Murray, Santa Barbara High School, Santa Barbara, California.

"The Place of Mathematics in the Secondary Education Report of the Joint Commission," Sara B. S. Rabourne, Fresno High School, Fresno, California.

"Problems in the Mathematics Department of the Small High School," Harry Renoud, Templeton Union High School, Templeton, California.

"Putting the Mathematics of Estimation and Measurement to Use in a Secondary Program," Charles Weidner, Associate Professor of Mathematics Education, Ohio State University, Columbus, Ohio.

On invitation by the American Mathematical Society, an International Congress of Mathematicians will be held in Cambridge, Massachusetts, in 1940.

The dates of the Congress have been fixed as September 4–12, 1940. Harvard University and the Massachusetts Institute of Technology will be the local hosts of the Congress.

An innovation will be conferences, somewhat after the pattern of recent international gatherings in Moscow for Topology and in Zürich for Probability. Each conference will be devoted to some field in which a vigorous advance has recently been made or is currently in progress. The purpose will be the exchange of information and opinion by specialists in the field, and the dissemination of new and important results among the mathematical public at large. This will be accomplished by a coördinated program of formal lectures and informal open discussion. Among the conferences will be one on Algebra, with Professor A. A. Albert as chairman, one on the Theory of Measure, Probability, and Allied Topics, with Professor Norbert Wiener as

chairman, one on Mathematical Logic with Professor H. B. Curry as chairman, and one on Topology with Professor Solomon Lefschetz as chairman.

Six sections are tentatively planned for the presentation of papers: (I) Algebra and Theory of Numbers; (II) Analysis; (III) Geometry and Topology; (IV) Probability, Statistics, Actuarial Science, Economics; (V) Mathematical Physics and Applied Mathematics; (VI) Logic, Philosophy, History, Didactics. The International Commission on the Teaching of Mathematics proposes to have a session in connection with the Congress.

These short papers will be preferably in one of the official languages of the Congress (English, French, German, and Italian), and will not exceed ten minutes in length.

Detailed information will be sent in due course to all members of the American Mathematical Society. Others interested in receiving information may file their names in the Office of the Society, and such persons will receive from time to time information regarding the program and arrangements.

Communications should be addressed to the American Mathematical Society, 531 West 116th Street, New York City, U.S.A.

THE ORGANIZING COMMITTEE

The Southern Intercollegiate Mathematics Association held its Sixth Annual Meeting in Jackson, Mississippi at Millsaps College, May 13.

The schools having won the right to participate in the Finals by virtue of winning the Preliminary Contests in their Respective Regions were: Mississippi Woman's College, Region I; Centenary College, Region II; Southern Methodist University, Region III; North Texas State Teachers College, Region IV.

Southern Methodist University was awarded the S.I.M.A. Cup for having the highest average in the Final Examinations.

The following students won the right to wear the S.I.M.A. key because they each made the highest grade in the Association in their respective subjects:

Preliminary Examination Winners: Merle Mitchell, S. M. U., algebra; Cleo White, Mississippi Woman's College, trigonometry; Mary Emma Fancher, Mississippi Woman's College, analytics; Nina Pearl Byrd, Mississippi Woman's College, calculus; Louise Tate, Mississippi Woman's College, comprehensive.

Final Examination Winners: Merle Mitchell, S. M. U., algebra; Julia Smith, S. M. U., trigonometry; Wilbur Teubner, S. M. U., comprehensive; William Tittle, North Texas State Teachers College, calculus; William E. Steger, Centenary College, analytics.

The following Member Professors attended: S. M. U., Mouzon, Huff, Palmquist, Wright; Centenary College, Hardin, Banks; Millsaps College, Mitchell, Van Hooks; North Texas State Teachers College, Hanson, Cooke; Mississippi Woman's College, Brown; Mississippi State Teachers College, Dearman; McMurry College, Tate.

JENNIE TATE, *Sec.-Treas. S.I.M.A.*

Professor J. H. Van Vleck of Harvard University gave a series of lectures at the Institut Henri Poincaré at the University of Paris, during the summer. He will also take part in a symposium on Magnetism at Strasbourg.

Professor G. D. Birkhoff of Harvard University has been appointed Exchange Professor to France for the second half of the academic year 1939–40.

The following men have been appointed Benjamin Peirce Instructors at Harvard University for the academic year 1939–40: Dr. Leon Alaoglu, Dr. Donald T. Perkins, and Dr. B. J. Pettis.

The following men have been appointed Instructors in Mathematics at Harvard University for the academic year 1939–40: M. H. Heins, A. D. Hestenes, D. T. McClay, E. N. Nilson, A. Spitzbart, and P. M. Whitman.

Association of Mathematics Teachers of New Jersey held its Sixty-sixth Regular Meeting at the State University of New Jersey in New Brunswick, on May 6. The program follows:

Conference Theme: "Education for Human Relationships in a Democracy."

JUNIOR HIGH SCHOOL—MRS. MARGARETTA BOULGER, *Presiding.*

10:15—"*Consumer Education and Mathematics,*" Mr. Hubert Risinger, Davey Junior High, East Orange, N.J.

11:15—"*Activity in Junior High School Mathematics,*" Mr. Francis P. Rice, Mt. Hebron Junior High, Montclair, N.J.

SENIOR HIGH SCHOOL

10:15—"*The Mathematical Value of*

SHORETOWN to its Citizens," Mrs. Grace B. Taylor, Manasquan High School, Manasquan, N.J.

11:00—"*College Preparatory Mathematics, Plus and Minus,*" Mr. Joseph P. Shuttlesworth, Summit High School, Summit, N.J.

11:45—"*A Mathematical Rating System for Football Teams,*" J. Whitney Colliton, Central High School, Trenton, N.J.

12:45—A luncheon was held at St. James Methodist Church to celebrate the twenty-fifth anniversary of the organization of the association.

The Association of Teachers of Mathematics of New York presented a series of twelve mathematics broadcasts over radio station WNYC last year. The schedule of broadcasts, all of which were given on Tuesday afternoons at 5:30, follows.

February 28—"Three Geometries," by Isidor Dressler of Cleveland H. S.

March 7—"Abel and Galois," by Harry Sitomer, New Utrecht H. S.

March 14—"The Story of Number," by Etta Greenberg, Irving H. S.

March 21—"Mathematics of the Ancients," by Dr. Nathan Lazar, Hamilton H. S.

March 28—"Mathematics, the Instrument of Everyday Living," by Dr. Harry Eisner, Manual Training H. S.

April 4—"Curious Facts of Mathematics," by Mannis Charosh, Lafayette H. S.

April 18—"Mathematics, the Mirror of Modern Civilization," by Dr. John A. Swenson, Andrew Jackson.

April 25—"Amazing Discoveries of Mathematics," by Jack Deutch, Jefferson H. S.

May 2—"True or False?" by Morris Hertzig, Haaren H. S.

May 9—"Zero and Infinity."

May 16—"The Story of Weights and Measures," by Francis Lepowsky, Flushing H. S.

May 23—"The Play of the Imagination in Mathematics," by Prof. W. D. Reeve, Columbia University.

The Minneapolis Mathematics Club carried through a worth-while program of meetings during the year 1938–39. The general theme for the year's programs was "The Curriculum."

On October 12, Miss Cutright, Assistant Superintendent of Schools, spoke on "Some Problems in the Field of Mathematics."

October 28, Professor W. E. Peik, Dean of the College of Education of the University of Minnesota, addressed the M. E. A. luncheon group on "Desirable Next Steps in the Mathematical Program."

A panel discussion on "The Present Curriculum in Use" took place December 6. Each of the six members of the panel spoke for ten minutes on these topics: "Consumer Mathematics," "Counseling in Mathematics at the Junior High School Level," "Elementary Mathematics in the Senior High School," "Advanced Mathematics," "Counseling for Senior High School Mathematics," and "What Can the Minneapolis Mathematics Club Do in the Solution of Problems Suggested in These Various Fields?"

Dean Peik spoke again on January 10 taking for his subject "Curriculum Issues of Secondary Schools with Special Reference to Mathematics."

"Differentiation of the Mathematics Curriculum" was the theme for the February meeting. Subjects of talks were "General Mathematics in the Tenth Grade," "Vocational Mathematics for Boys," and "The Counselor Looks at Mathematics."

"Summarization and Implementation of the Curriculum" was the subject of the March discussion meeting.

On May 5 and 6, the Minneapolis Mathematics Club met with the Minnesota Conference of Teachers of Mathematics at the University of Minnesota. The program follows:

FRIDAY, MAY 5, 10:00–11:45

General Theme: "The Place of Vocational Mathematics in the Secondary School Mathematics Curriculum."

Chairman: P. A. Samuelson, Roosevelt High School, Minneapolis.

Speakers: Dr. A. M. Field, Professor of Agricultural Education, University of Minnesota; R. T. Craigo, Director of Day School, Dunwoody Institute, Minneapolis; R. C. Behn, Examiner of the Classified Service, University of Minnesota.

FRIDAY LUNCHEON, MAY 5, 12:15–1:30

Chairman: Dr. W. E. Peik, Dean, College of Education, University of Minnesota.

Speaker: Dr. L. J. Brueckner, Professor of Education, University of Minnesota, "Trends in Instruction in Mathematics."

FRIDAY, MAY 5, 3:00–4:30

General Theme: "The Secondary School Mathematics Curriculum from the Point or View of the Administrator."

Chairman: Dr. W. S. Carlson, Director of Training School, University of Minnesota.

Speakers: F. J. Herda, Principal, Senior-Junior High School, Alexandria; D. F. Dickerson,

Superintendent of Schools, Winona; Waldemar Hagen, Director of Personnel Department, University High School, University of Minnesota.

SATURDAY, MAY 6, 10:00–11:45

General Theme: "The Secondary School Mathematics Curriculum in Relation to Higher Education."

Chairman: W. B. Gundlach, Mathematics Teacher, Rochester.

Speakers: Dr. Raymond W. Brink, Professor of Mathematics, University of Minnesota; Dr. Malcolm S. MacLean, Director, General College, University of Minnesota; Dr. John G. Rockwell, Commissioner of Education, State of Minnesota.

SATURDAY LUNCHEON, MAY 6, 12:15–1:30

Chairman: Dr. A. B. Caldwell, Deputy Commissioner of Education, State of Minnesota.

Speaker: W. B. Gundlach, Chairman of State Committee for the Reorganization of Mathematics, "Suggested Program for the Reorganization of Junior High School Mathematics."

Professor Lester R. Ford, Chairman of the Department of Mathematics, Armour Institute of Technology, discussed "Quadratic Equations" at the April meeting of the Men's Mathematics Club of Chicago.

Dr. Ralph W. Tyler, Chairman of the Department of Education and Chief Examiner of the Board of Examinations, University of Chicago, spoke on "Evaluating Student Progress in Mathematics."

The May meeting was featured as Past Presidents' Night. The principal speaker was Professor John R. Clark, Teachers College, Columbia University, one of the past presidents.

NEW BOOKS

The Concepts of the Calculus. A Critical and Historical Discussion of the Derivative and the Integral. By Carl B. Boyer. Columbia University Press, 1939. 346 pp. Price, $3.75.

On rare occasions the literature of scientific thought becomes notably enriched by an outstanding work such as Dantzig's *Number: The Language of Science* or E. T. Bell's *Men of Mathematics.* Another such welcome occasion is provided by the appearance of Dr. Boyer's undeniably scholarly and carefully documented study. It is a thoroughly adequate examination of the origins and evolution of the fundamental notions underlying modern analysis. It is far more, however, than a mere chronological account or a compelling historical narrative; it is a searching analysis and critical evaluation of the well-sources of certain crucial ideas, and of their subsequent transformations, reappearances and ultimate refinements. As such, it must be regarded as a genuine contribution, for which both laymen and scholars owe the author a debt of gratitude.

The book is doubly welcome: in addition to an agreeable lucidity of style and singular clarity of thought, the interpretation is born of that rare combination of competent mathematical insight and sympathetic historical understanding. Moreover, an adequate history of the calculus needed to be written, despite the wealth of material scattered here and there. Most discussions of the development of the calculus terminate with the work of Newton and Leibniz; most of them are content to refer to the work of Eudoxus, Archimedes, Cavelieri, Roberval, Kepler, Barrow, Wallis and others simply as "forerunners" of the calculus. The author, however, not only brings the story down to modern times to include the rigorous refinements of Cauchy, Weierstrass and Cantor, but throughout the entire study he retraces with fine sensitivity and rare insight the delicately interwoven threads of such subtle concepts as limit, infinite, continuum, real number, incommensurable and the like.

The history of the calculus may be characterized by three significant considerations. In the first place, the calculus cannot be regarded as the invention of any one or more individuals. As Sarton has aptly pointed out, "admirers of great men often make the mistake of giving them credit for the endless consequences of their discoveries, consequences which they could not possibly foresee." Thus Newton and Liebniz could no more anticipate Cauchy's notion of the differential or Weierstrass' concept of the limit than Eudoxus or Archimedes could have foreseen Newton's fluxions or the derivative of Leibniz. In short, modern analysis would have been unthinkable without the cumulative filiation of ideas built up into a continuous development by many contributors.

In the second place, as commonly happens in the field of scientific thought, so in the case of the calculus, the clear and adequate understanding of the fundamental notions underlying the subject were not achieved until comparatively late in its development. If we consider the recognition of the fundamental inverse relationship

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^n f(x_i)\Delta x_i$$

as constituting the invention of the calculus, then it is clear that it required over a hundred years of further effort before the achievement of the above niceties was effected.

Finally, the development of these two concepts, the derivative and the integral, from their incipiency in sensory experience and intuition to their ultimate elaboration into logically rigorous abstractions, affords a striking example of a protracted struggle for the emancipation of abstract concepts from the earth-bound origins from whence they sprang.

All three of these considerations are clearly emphasized in the present work, as may be seen from the following sequence of chapters:

I. Conceptions in Antiquity
II. Medieval Contributions
III. A Century of Anticipation
IV. Newton and Leibniz
V. The Period of Indecision
VI. The Rigorous Formulation

This is followed by an admirably trenchant concluding summary. An excellent bibliography of some 500 references is also given. The typography is more than satisfactory, and the work is unusually free of errors.

Over and above his unquestioned mastery of the subject, the author has drawn freely upon his unusually rich background in the fields of the history of science and philosophy. The book fills a long-felt need. No teacher of mathematics can afford not to read it.

W. L. Schaaf

Mathematics in Action. Book One. By Walter W. Hart and Lora D. Jahn. D. C. Heath and Company, 1939. 344 pp. Price, $.88.

The teacher who uses this book for the seventh grade will have enough material for drill on skills, for the book has plenty of exercises. For example, percentage, that early junior high hurdle to be jumped, has three units devoted directly to it, with simple good explanations and many problems.

Moreover, the authors have been successful in producing a text for a "practical, socialized course" as they claim. The socialized mathematics topics include thrift, budgets, installment buying, banking, transportation, discount, cost and profit. Some of these are treated in separate units, some as applications of arithmetical principles.

Introductory work in geometry occurs in Unit I, entitled "Commonly Used Geometrical Figures"; Unit IX, "Straight Lines and Angles"; Unit XI, "Circles and Their Uses"; and Unit XII, "Triangles—Parallelograms— Trapezoids"; that is, four units out of the fifteen in the book. This should be helpful to junior high school teachers who face a change from formal arithmetic to general mathematics and who are puzzled as to what to do about geometry on that level.

E.W.

New Practical Mathematics. Revised Edition. By N. J. Lennes. The Macmillan Company, 1939. 426 pp. Price, $1.32.

The growth of the general mathematics movement has brought out a crop of new ninth grade mathematics textbooks, many of them good, modern, and filled with considerations and problems drawn from life. Here is the revised edition of one, first published in 1936, which has the advantages of three years maturing and development as the result of classroom use together with the opportunity to rewrite the book in the light of what has happened to it in those three years.

It combines training in fundamental processes and skills with mathematics-in-life in a two-part arrangement. Part I, entitled "A Restudy of Elementary Mathematics," includes these chapter topics: Whole Numbers and Decimals, Common Fractions, Equations, Percentage, Simple Applications of Percentage, Short Cuts, Graphs, and Measuring.

Part II, called "Solving Practical Problems," contains most of the materials that make this book different from others in its class. Again the chapter titles give a good idea of what is in this section. They are: Cost of Owning a Car, Cost of Owning a House and Other Problems, Owning a Farm, Insurance, Taxes, Miscellaneous Home Problems, Installment Buying, Buying, Preparing, and Serving Food, Household Accounting, Buying Lumber; Problems for Carpenters, Excavating; Cement Work, Useful Geometric Figures, Shop Problems and Other Problems, Farm Problems, and The Grade Curve and Other Graphs.

A carefully planned social studies slant may be observed. It is mathematics less for its own sake and its specialized use than for the sake of the average citizen's need of it all through life.

It is an excellent book for those students who may take only one course in high school mathematics or who need a year for catching up to themselves before plunging into college preparatory algebra and geometry.

E.W.

Mathematics in Daily Life. By Eugene H. Barker and Frank M. Morgan. Houghton Mifflin Company, 1939. 432 pp. Price, $1.32.

Other textbooks besides this ninth grade general mathematics have used a typical American family as a basis of motivation for junior high school mathematics. But the "Baxters" are a particularly amiable bunch whose experiences and earnest conversations about topics mathematical should reassure the most skeptical child as to the value of the subject. Even undemonstrative geometry becomes involved in sports as well as in finding out whether the family is getting its money's work of purchased materials.

The Table of Contents shows that chapter titles emphasize the social aspects of mathematics. Indeed, "The Organization of Business," "Paying the Government's Bills," and "Business Related to the Household" might be chapters in a social studies text. However, they include such topics as profit and loss, stocks and bonds, dividends, taxes, budgets and keeping accounts, with plenty of problems and many applications of mathematical principles and skills such as, for instance, working with per cents and the fundamental processes of arithmetic.

Specifically, here is a textbook for general mathematics, easy and pleasant to read, which shows the application to life of the mathematics taught. The selection of topics is practical and typical of textbooks of this kind.

E.W.

Understanding Our Environment. By John C. Hessler and Henry C. Shoudy. Benj. H. Sanborn and Company, 1939. 661 pp. Price, $1.80.

Environment is a key word in junior high

school science, and this ninth grade book has been properly named, for it is a book by which a student may not only learn to understand man's environment, but how environment affects man and how he may deal with it.

Grouped within nineteen Units is more than a good coverage of the topics usually found in general science textbooks. There are sixty-six well developed experiments with directions, lists of materials, and questions under the headings of Observations and Conclusions, so that a separate laboratory manual is unnecessary.

Each Unit is broken up into numbered divisions called Problems. The titles of all of these parts are in question form, a rather catachismic array as one looks through the book. This is calculated to emphasize and stimulate the inquiring spirit. Each Unit has its teaching aids of Preview Questions, Tests, Questions for Discussion, Review Questions, and list of Selected Readings.

Throughout the book are constant applications of facts and principles to practical living. Health, consumer problems, the home environment, and many other immediate concerns of life are handled in a way to make more able homemakers and citizens.

E.W.

Laboratory Experiments and Workbook. To accompany Black and Davis' *Elementary Practical Physics.* By Newton Henry Black and Elbert Cook Weaver. The Macmillan Company, 1938. 290 pp. Price, $1.00.

The May issue of The Mathematics Teacher carried a review of Black's *Laboratory Experiments in Elementary Physics,* bound in cloth, with the comment that many high-school teachers and students would prefer a large, paper-covered workbook type of manual. Since the publication of the May number, Black and Weaver's workbook laboratory manual has come to the reviewer's desk.

The combination laboratory manual and workbook, which fulfills every reasonable specification for both purposes, has been prepared with imagination and skill. Each unit has an introductory page with a symbolic artist's drawing, combined with text material giving a preview of the content of the unit and its applications to life. A page of suggestive questions follows.

Since this manual and workbook is made to be written in, no other notebook is necessary. The experiments are so arranged that besides directions and diagrams, there is plenty of space provided for writing data, results, and conclusions. Following the body of experiments for the unit are several pages of exercises and a review of the unit. Approximately half the book is devoted to these workbook type exercises. In addition an unbound packet of tests comes with the manual.

This is a manual to simplify the mechanics of teaching and to eliminate some of the less useful work of students. It has the classical correctness of other books of the series of which it is a part and will add good ideas and sparkle to any high-school physics course.

E.W.

*The Nature of Proof.** By H. P. Fawcett. Pp. xi, 146. 1938. Thirteenth Yearbook of the National Council of Teachers of Mathematics. (Columbia University, New York)

This is a fascinating book which can be most warmly recommended to all teachers of elementary geometry. Its title disguises the fact that it is mainly the record of an experiment, on lines which no doubt have suggested themselves to most teachers; but I have not hitherto seen an account of any similar experiment carried out so carefully and over such a long period of time. The first chapter discusses the values which are claimed for demonstrative geometry, the second chapter describes the experimental conditions and data. Fifty pupils from the ninth, tenth and eleventh grades, with an age spread from 13 to 18, the bulk being of age 16, were divided into two classes A and B; for four forty-minute periods a week for sixty-eight weeks class B took an ordinary course in formal geometry, while class A was used as the experimental material; the division was more or less a random section, as the tables comparing ages, grades and even intelligence scores for the two groups show, though since intelligence tests have convinced the reviewer that he is practically an imbecile, he is perhaps unfairly prejudiced against such tables. The author is careful to state that the superficial inference that class B could be taken as a control is unsound, for reasons which will be clear to any teacher.

Class A, then, arriving for its first period, complete with notebooks, pencils, rulers and compasses, waiting to be told to obtain copies of Messrs. So and So's *New Geometry*, was not unnaturally surprised when the teacher remarked that, as there was no great hurry for the geometry, they might discuss the practical proposition "that awards should be granted for outstanding achievement in the school." After a vigorous discussion from which nothing much emerged, one pupil remarked, "Most of this trouble is caused by the fact that we don't know what we mean by 'awards' or by 'outstanding

* Reprinted from *The Mathematical Gazette*, XXIII (May, 1939), pp. 238–239.

achievement.' " Four weeks were spent on various discussions, and pupils and teacher then agreed on the need for precise definitions and explicit recognition of assumptions; they also decided that definitions and assumptions are not sent down ready-made from Heaven, but must be the result of group agreement; and discussions on "What is an aristocrat?" "What is the labor class?" "What is an obscene book?" suggested as a corollary that such group agreement is difficult to obtain "in situations which cause one to become excited." Various proposals for further discussions were made and rejected as likely to lead to emotional complications, and a suggestion from the teacher resulted in an agreement to "build a theory about the space in which we live." The main lines of the remainder of the course will now be apparent; the collection and classification of undefined terms, definitions, assumptions, the beginnings of formal proof, the slow but thorough construction by the pupil of his own textbook. There is much in the detailed account which will be of value to the teacher, many points which he may find can be worked into his own more pedestrian geometry lessons, much food for thought and experiment on the conduct and aims of a geometry course in the more severely circumscribed conditions under which he normally works.

We are also given "evaluations" of the course written by observers who included students training in a college of education, by the pupils and by their parents. These seem to show that the class-discussions were real, a valuable piece of evidence, since it is very difficult for the teacher himself to be sure that the class is really discussing the subject and not merely following his leads. The pupils themselves evidently enjoyed the course and profited by it; many of them assert, apparently sincerely, that they had lost the dread of geometry which they had felt before the course began. The parents agree, in the main, that the course had been of real value in improving the ability of the pupils to think critically, though some, not unnaturally, feel that the tendency to think critically produced cynicism and quibbling. Finally, from essays on "The evolution of proof" written by the students, the teacher, with the assistance of the English teacher, selected two of the best, two of average quality and two of the worst, and these six are printed in an Appendix. These are as interesting as anything else in the book and are themselves sufficient evidence of the value of the course. One small point: to disparage Euclid because, many centuries after his death, people were foolish enough to use his *Elements* as a textbook for young children, is illogical; I suspect that the reason that some of the essays incline to this disparagement is that among the books used by the students in preparing these essays is one whose stimulus is unfortunately dangerous because of a false historical perspective.

T.A.A.B.

A Picture Dictionary for Children. By Garnette Watters and S. A. Courtis. Grosset & Dunlap, 1939. 478 pp. Price, $1.00.

At one time, a well-known progressive school had on its applications for admission a question something like this: Did the child learn to read *spontaneously?* To many persons the idea of a child learning to read spontaneously (possibly that means self-taught) is a mystifying one. However, with this picture dictionary, an energetic, curious child could go a long way in teaching himself to read.

This book for children in the first three grades of school has 2154 basic words and 2678 variants. Of these 1200 are illustrated by artists' drawings. Some words, for instance *not*, cannot be clearly illustrated by a picture, and it is only such words that are not accompanied by a drawing. A word is not defined formally in the manner of adult dictionaries, but is used in simple sentences. It is also given three ways: printed in the type face of the book, in hand writing, and in manuscript writing. If the word has more than one syllable, it is printed with the syllable divisions shown.

This dictionary has been developed and tested in the Hamtramck Public Schools. There experiments showed that pupils using it developed 2.09 times as fast in reading and 4.35 times as fast in spelling as those who did not.

A teacher handling learning-to-read children can help them inestimably by having this book, the first of its kind, where they can refer to it, whether purposefully in looking for specific words, or in the apparently idling manner of children, who so often learn much when they do not realize that they are in a learning situation.

E.W.